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Abstract

This paper presents an open economy DSGE model, which is estimated on a euro area data set using Bayesian techniques. An attempt is made to impose stochastic assumptions which are consistent with observed trends. In particular we allow for a unit root in technology which allows us to work with actual growth rates. In addition we respect the long run equilibrium constraints implied by the model. The model is compared to a VECM in order to detect weaknesses in the specification. A full Bayesian IRF analysis is performed with a detailed sensitivity analysis of the IRF shape versus model coefficients.

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Introduction

This paper presents an estimated open economy DSGE model for the euro area. Optimising consumers and firms decide over consumption and investment of domestic and foreign goods and domestic firms sell to the home and foreign market. Behavioural trade relationships are derived under the assumption that domestic and foreign goods are imperfect substitutes. The essential open economy features are that foreign prices, income and interest rates are given exogenously (and generated by a VAR). Concerning international financial markets it is assumed that interest parity holds up to a risk premium which is a function of net foreign debt. There are nominal rigidities in the labour market and the market for domestic goods, exports and imports. In the open economy context this implies that there is incomplete pass through of the exchange rate.

The model is estimated on euro area data using Bayesian estimation techniques. The procedure is implemented using the DYNARE toolbox for MATLAB. Smets and Wouters (2003) and Ratto et al (2004), for example have used this approach to estimate closed economy versions of the new Keynesian DSGE model.

Apart from the fact that the open economy extension adds additional shocks and transmission mechanisms to the model we also deviate from previous exercises by allowing stochastic trends. This allows us to formulate the model in growth rates and certain stationary ratios of actual data. These ratios are derived from the steady state of the model along the non stochastic technology trend. The standard specification of the model implies in particular that the nominal investment and consumption to GDP ratio, the nominal wage share, the employment rate and the nominal trade balance to GDP ratio will be constant along the steady state. A careful check of the data reveals that some of these ratios depart significantly from the long run model prediction over the sample period (1980q1 to 2003q4). In particular this is true for the nominal wage share which shows a significant downward trend and for the employment rate which exhibits non stationary fluctuations. This makes it necessary to formulate the model in such a way that it can meet these requirements. Concerning the wage share we consider two hypotheses, a rising mark up vs. a decline in overhead labour. It turns out that the latter hypothesis fits the data better.

In the estimation we devote special attention to the set of parameter coefficients ensuring saddle path stability of our forward looking model. When the degree of complexity of the model structure and its parameterization increases, it might not be trivial to know a priori the set of model coefficients assuring the rank condition for the solution of forward looking components. The paper presents a simple mapping procedure, based on the filtering of Monte Carlo samples of model coefficients within prior bounds. As shown below this procedure is useful in restricting the priors for the coefficients. The case discussed in the paper is the relationship between coefficients in the import/export equations (adjustment costs and share of forward looking behaving agents), the persistency of nominal interest rates in the Taylor rule and the share of forward looking behaving firms in the mark-up factor. More specifically it turns out that combinations of high import/export rigidities and small fractions of forward looking firms in import/export make the system unstable.

The fit of the model is evaluated by comparing it to a VECM with cointegrating constraints consistent with those imposed by the DSGE model and estimated on the same information set.

We compare RMSE's for one and four period ahead projections of both types of models. Our general finding is that the VECM outperforms the DSGE model for one period projections while the four step ahead prediction errors tend to be much closer.

The remainder of the paper addresses directions in which the fit of the model could be improved. We are currently pursuing two alternative directions:

1. First, we systematically analyze the DSGE prediction error versus model coefficients, in order to detect which of them mostly drive the fit of each observed series, as well as possible 'trade-off' effects, i.e. if the good fit of two or more different series is associated with disjoint subsets of values of a given coefficient.
2. A second avenue that we explore is checking for the presence of seasonal effects. Seasonality in the data could potentially explain the one and four step prediction errors. Preliminary data analysis in fact suggests the presence of seasonality in the data.

The paper is structured as follows. In the first section the model is described together with the long run constraints implied by the model. The second section gives details about the estimation (in particular some discussion on the selection of parameter ranges consistent with the Blanchard Kahn, 1980, stability conditions) and presents the estimation results. Section three evaluates the DSGE by comparing the fit of the VECM and the DSGE model at various forecast horizons as well as a detailed sensitivity analysis of the RMSE's of each single observed series versus model coefficients. A final section presents impulse responses and provides sensitivity analysis.

1. The Model

The model belongs to the class of (stochastic) general equilibrium models that have been used for macroeconomic analysis both in the closed and open economy literature. We consider a small open economy which faces an exogenous world interest rate, world prices and world demand. The domestic and foreign regions produce a continuum of differentiated goods. The goods produced in the home country are imperfect substitutes for goods produced abroad.

1.1 Households:

The household sector consists of a continuum of households $i \in (0,1)$. They decide about consumption, labour supply and asset allocation. Each household supplies a specific variety of labour in a monopolistically competitive labour market, i.e. the household sector sets the wage given the demand curve for labour. Nominal rigidity in wage setting is introduced by assuming that the household faces adjustment costs for changing wages. These adjustment costs are borne by the household. The household decides about four types of assets, domestic and foreign nominal bonds, stocks of domestic companies and cash balances. The Lagrangian of this maximisation problem is given by

$$Max \quad U_0^i = E_0 \sum_{t=0}^{\infty} \beta^t \left(U(C_t^i) + V(1-L_t^i) + Z(M_t^i / P_t) \right)$$

$$-\sum \lambda_t \beta^t \left(\begin{array}{l} \frac{PC_t}{P_t} C_t^i + \frac{M_t^i}{P_t} + \frac{B_t^i}{P_t} + \frac{E_t B_t^{iF}}{P_t} + \frac{PC_t Q_t K_t^i}{P_t} - \frac{M_{t-1}^i}{P_t} - \frac{(1+i_{t-1})B_{t-1}^i}{P_t} - \frac{(1+i_{t-1}^F)B_{t-1}^{iF}}{P_t} \\ -\left(\frac{(1+div_t)PC_t Q_t K_{t-1}^i}{P_t} \right) - \frac{W_t^i}{P_t} L_t^i + \frac{\gamma_w L_t^i}{2} \left(\frac{\Delta w_t^i}{w_t} \right)^2 + TAX_t \end{array} \right)$$

The utility function is additively separable. For the model economy to attain a steady state we assume log utility for consumption and CES for leisure. In addition we allow for habit persistence in consumption and leisure. Thus temporal utility is given by.

(2a) $$U(C_t^i) = \exp(e_t^C) \log((1+habc)C_t^i - habcC_{t-1})$$

and

(2b) $$V(1-L_t^i) = \exp(e_t^L) \frac{\omega}{1-\kappa} ((1+habl)(1-L_t^i) - habl(1-L_{t-1}^i))^{1-\kappa} \quad \text{with } \kappa > 0,$$

where e_t^C and e_t^L are autocorrelated shocks to preferences:
$e_t^C = \rho^C e_{t-1}^C + \varepsilon_t^C$, $\varepsilon_t^C \sim N(0, \sigma^C)$ and $e_t^L = \rho^L e_{t-1}^L + \varepsilon_t^L$, $\varepsilon_t^L \sim N(0, \sigma^L)$
The consumption index is itself an aggregate over varieties of domestic and foreign goods which are imperfect substitutes. These preferences are expressed by a nested CES utility function. It is assumed that households, firms and the government have identical preferences over domestic and foreign varieties in order to facilitate aggregation. The sub utility functions are described in more detail in section 1.4 below.

The household decides about consumption, asset allocation and the supply of labour (or more correctly about wages) and real money holdings[1]. The first order conditions of the household (FOCs) with respect to consumption and financial wealth are given by the following equations:

(3a) $$\frac{\partial U_0}{\partial C_t^i} => U_{C,t}^i - \lambda_t \frac{PC_t}{P_t} = 0$$

(3b) $$\frac{\partial U_0}{\partial B_t^i} => -\lambda_t + {}_t\lambda_{t+1} \beta (1+i_t) \frac{P_t}{{}_tP_{t+1}} = 0$$

(3c) $$\frac{\partial U_0}{\partial B_t^{iF}} => -\lambda_t + {}_t\lambda_{t+1} \beta (1+i_t^F) \frac{P_t}{{}_tP_{t+1}} \frac{{}_tE_{t+1}}{E_t} = 0$$

[1] With an interest rate rule as specified below, an optimality condition for money would only determine the desired money holdings of the household sector without any further consequence for the rest of the economy. For that reason any further discussion on money demand is dropped here.

(3d) $$\frac{\partial U_0}{\partial K_t^i} => -\lambda_t \frac{PC_t Q_t}{P_t} + {}_t\lambda_{t+1}\beta \frac{\left((1+div_t)\, {}_tPC_{t+1}\, {}_tQ_{t+1}\right)}{{}_tP_{t+1}} = 0$$

(3e) $$\frac{\partial U_0}{\partial M} => \frac{M_t^i}{P_t} - Y_t^i R_t^{\zeta} = 0$$

From the FOC we obtain the following arbitrage equations

(3b') $$(1+i_t)\frac{P_t}{{}_tP_{t+1}} = (1+i_t^F)\frac{{}_tE_{t+1}}{E_t}\frac{P_t}{{}_tP_{t+1}}$$

(3d') $$(1+i_t)\frac{P_t}{{}_tP_{t+1}} = \left((1+div_t)\frac{{}_tQ_{t+1}}{Q_t}\right)\frac{{}_tPC_{t+1}}{PC_t}\frac{P_t}{{}_tP_{t+1}}$$

The first arbitrage condition requires that the return from a domestic bond is equal to the return from a foreign bond expressed in the domestic currency. The second arbitrage condition requires that the return from equity, i. e. dividends plus changes in the value of the capital stock plus changes in the price of capital goods is equal to the nominal interest rate.

Workers have market power in the labour market, because they offer services, which are imperfect substitutes to services offered by other workers. That means aggregate labour demand of firms is a composite of labour supplied by individual workers. Total employment in production is characterised by a CES function

(4a) $$L_t = \left[\int_0^1 L_t^{i\,\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}} \text{ with } \theta > 1$$

where the parameter θ determines the degree of substitutability between labour supplied by individual households. Corresponding to the CES aggregator there exists a wage index

(4b) $$W_t = \left[\int_0^1 W_t^{i\,1-\theta}\right]^{\frac{1}{1-\theta}}$$

This yields a labour demand equation as perceived by household *i*

(4c) $$L_t^i = \left(\frac{W_t^i}{W_t}\right)^{-\theta} L_t$$

In a monopolistic labour market the elasticity of substitution between different types of labour is important for determining the mark-up of wages over the equilibrium wage. This elasticity is defined by

(4d) $\frac{\partial L_t^i}{\partial W_t^i} = -\theta \left(\frac{W_t^i}{W_t} \right)^{-\theta} L_t \frac{1}{W_t^i} = -\theta \frac{L_t^i}{W_t^i}$.

Now the wage setting rule can be derived taking derivatives of the Lagrangian w.r.t. wages. Using symmetry: $W_t^i = W_t$ and neglecting second order terms allows us to write

(5a) $\frac{\partial U_0}{\partial W_t^i} => V_L = \lambda_t \left[\frac{(1-\theta)}{-\theta} \frac{W_t}{P_t} - \gamma_w \pi_t^W \right] + \lambda_{t+1} \beta \gamma_{w\,t} \pi_{t+1}^W$,

where π_t^W is the growth rate of nominal wages. This can be reformulated as a wage setting rule

(5b) $\pi_t^w = \frac{1}{\gamma_w} \left[\frac{C_t}{\omega (1-L)^\kappa} - (1 - mup^w) \frac{W_t}{P_t} \right] + \frac{1}{R_t}{}_t \pi_{t+1}^w$ with $mup^w = -\frac{1}{\theta}$

where wage inflation is determined by the gap between the reservation wage and the real wage adjusted for a wage mark up. The forward looking nature of wage setting is reflected by the forward wage inflation term. This formulation generalises the neoclassical labour supply model along two dimensions. First, by introducing convex wage adjustment costs ($\gamma_w > 0$), workers want to smooth wage adjustments, taking into account current and future expected labour market conditions. Second, because workers offer services which are imperfect substitutes to services offered by other workers, they can demand wages which are above their reservation wage[2]. The reservation wage is the marginal value of leisure, divided by the marginal utility of consumption. That means for a given utility of leisure the reservation wage increases with a decline in the marginal utility of consumption that an additional unit of labour can buy.

1.2 Firms:

There are n^d firms indexed by *j*. Each firm produces a variety of the domestic good which is an imperfect substitute for varieties produced by other firms. Because of imperfect substitutability, firms are monopolistically competitive in the goods market and face a demand function for goods. Domestic firms sell to private domestic households, to other firms the government and to exporting firms. All demand sectors have identical preferences across varieties. The demand function for firm *j* consistent with preferences (see section 1.4 for a more detailed description) is given by

(6) $Y_t^j = \frac{1}{n^d} \left(\frac{P_t^j}{P_t} \right)^{-\frac{1}{\tau_t}} \left[\left(C_t^D + G_t^D + I_t^D + EX_t \right) \right]$

In what follows it is assumed that firms influence the demand for domestic goods with their pricing decision, however, they are small with respect to the total market and therefore take as

[2] Notice in the limiting case of perfect substitutability ($\lim \theta \to \infty$), the mark up approaches zero.

given P_t , PC_t , C_t, G_t, I_t and EX_t. Output is produced with a Cobb Douglas production function

(7) $$Y_t^j = (ucap_t^j K_t^j)^{1-\alpha} (L_t^j - LO_t^j)^\alpha U_t^\alpha = (ucap_t^j K_t^j)^{1-\alpha} [L_t^j \cdot (1 - LOL_t^j)]^\alpha U_t^\alpha$$

with capital and labour minus overhead labour as inputs. Firms can also decide about the degree of capacity utilisation. The level of technology is subject to autocorrelated technology shocks (e_t^U) and follows a random walk with drift g^U

(8a) $$\log(U_t) = g^U + \log(U_{t-1}) + e_t^U .$$

(8b) $$e_t^U = \rho^U e_{t-1}^U + \varepsilon_t^U , \qquad \varepsilon_t^U \sim N(0, \sigma^U)$$

We allow LOL (overhead to labour share) to follow an AR(1) process:

(8c) $$LOL_t^j = \rho^{LOL} LOL_{t-1}^j + \varepsilon_t^{LOL} \ , \ \ \varepsilon_t^{LOL} \sim N(0, \sigma^{LOL})$$

The objective of the firm is to maximise the present discounted value of its cash flows. The link to the household sector is as follows. Domestic firms are owned by domestic households. All investment is equity financed and the firms pay dividends to the household sector. Dynamic considerations enter the problem of the firms because firms faces quadratic costs of changing capital, employment and prices. Finally firms must also choose the optimal level of capacity utilisation.

(9)

$$\begin{aligned}
Max\ V_0^{r\,j} &= E_0 \sum_{t=0}^{\infty} d^t \frac{\left[P_t^j(.)Y_t^j - W_t L_t^j - PC_t I_t^j - adc^P(P_t^j) - adj^K(K_t^j, I_t^j) - adj^L(L_t^j) - adj^{CAP}(ucap_t^j)\right]}{P_t} \\
&- \sum \eta_t^j d^t \left[Y_t^j - (ucap_t^j K_t^j)^{1-\alpha} (L_t^j (1 - LOL_t^j) U_t)^\alpha\right] \\
&- \sum \mu_t^j d^t \left[K_t^j - I_t^j - (1-\delta) K_{t-1}^j\right]
\end{aligned}$$

where $d^t = \prod_{l=0}^{t} \left(\frac{1}{1 + r_l + rp_l} \right)^l$ is the discount factor, which consists of the short term interest rate and a risk premium (*rp*). The risk premium can be subject to random shocks and generated by the following autoregressive process

(10) $$rp_t = \rho^{rp} rp_{t-1} + (1 - \rho^{rp}) rp + \varepsilon_t^{rp} , \qquad \varepsilon_t^{rp} \sim N(0, \sigma^{rp})$$

For adjustment costs we choose the following convex functional forms

(11)

$$adj^L(L_t^j) = \exp(e_t^W)W_t(L_t^j + \frac{\gamma_L}{2}\Delta L_t^{j2})$$

$$adj^P(P_t^j) = \frac{\gamma_P}{2}\Delta\pi_t^{j2} \quad ,with \quad \pi_t^j = P_t^j / P_{t-1}^j - 1$$

$$adj^K(K_t^j, I_t^j) = PC_t\left(\frac{\gamma_K \exp(e_t^I)}{2}\frac{I_t^{j2}}{K_{t-1}} + \frac{\gamma_I}{2}\Delta I_t^{j2}\right)$$

$$adj^{CAP}(ucap_t^j) = a_1(ucap_t^j - ucap^*) + a_2(ucap_t^j - ucap^*)^2$$

The firm determines labour input, the capital stock, capacity utilisation and prices optimally in each period given the technological and administrative constraints as well as demand conditions. The first order conditions are given by:

(12a) $$\frac{\partial V_0^{r\,j}}{\partial L_t^j} \Rightarrow \left(\alpha\frac{Y_t^j}{L_t^j}(1+LOL_t^j)\eta_t^j + \frac{\gamma_L}{R_t}({}_tL_{t+1}^j - L_t^j) - \gamma_L(L_t^j - L_{t-1}^j)\right) = \frac{W_t}{P_t^j}(1+\varepsilon_t^W)$$

$$\varepsilon_t^W \sim N(0, \sigma^W)$$

(12b) $$\frac{\partial V_0^{r\,j}}{\partial I_t^j} \Rightarrow \frac{PC_t}{P_t}\gamma_K(1+e_t^I)\frac{I_t^j}{K_{t-1}^j} + \gamma_I(\Delta I_t^j - \frac{1}{R}\Delta_t I_{t+1}^j) = \mu_t^j - \frac{PC_t}{P_t}$$

$$e_t^I = \rho^I e_{t-1}^I + \varepsilon_t^I,\ \varepsilon_t^I \sim N(0, \sigma^I)$$

(12c) $$\frac{\partial V_0^{r\,j}}{\partial K_t^j} \Rightarrow (1-\alpha)\frac{Y_t^j}{K_t^j}\eta_t - ((a_2 - a_1) + (a_1 - 2a_2)ucap_t^j + a_2 ucap_t^{j2}) =$$

$$\mu_t^j - (1 - r_t - rp_t - \delta)_t\mu_{t+1}^j$$

(12d) $$\frac{\partial V_0^{r\,j}}{\partial ucap_t^j} \Rightarrow ((a_1 - 2a_2) + 2a_1(ucap_t^j)) = (1-\alpha)\frac{Y_t^j}{K_t^j ucap_t^j}\eta_t^j$$

(12e) $$\frac{\partial V_0^{r\,j}}{\partial Y_t^j} \Rightarrow \eta_t^j = 1 - (\tau^0 + \tau^1(Y_t - Ypot_t) + e_t^\tau) + \gamma_P\left[\beta\,{}_t\pi_{t+1}^j - \pi_t^j\right]$$

Firms equate the marginal product of labour, net of adjustment costs, to wage costs. Wage costs include an autocorrelated wage cost shock. This should be seen as shocks to administrative burdens related to current employment. As can be seen from the left hand side of equation (12a), the convex part of the adjustment cost function penalises in cost terms accelerations and decelerations of changes in employment. Equations (12b-d) jointly determine the optimal capital stock and optimal capacity utilisation. The firm equates the marginal product of capital to the rental price of capital, adjusted for capital costs. The firm also equates the marginal product of capital services (*K*ucap*) to the marginal cost of capacity utilisation. Equation (12e) defines the mark up factor as a function of the elasticity of substitution and changes in inflation. We follow Smets and Wouters and allow for additional backward looking elements by assuming that a fraction (*1-sfp*) of firms keep prices fixed at the t-1 level. This leads to the following specification:

(12e') $$\eta_t^j = 1 - (\tau^0 + \tau^1(Y_t - Ypot_t) + e_t^\tau) + \gamma_P\left[\beta(sfp^*{}_t\,\pi_{t+1}^j + (1-sfp)\pi_{t-1}^j) - \pi_t^j\right] \quad 0 \le sfp \le 1$$

with

(12e'') $e_t^{\tau} = \rho^{\tau} e_{t-1}^{\tau} + \varepsilon_t^{\tau}$, $\varepsilon_t^{\tau} \sim N(0, \sigma^{\tau})$.

1.3 Government sector:

Fiscal Policy:
The government sector and fiscal policy is treated in a rather rudimentary fashion. The share of government purchases in nominal terms

(13a) $$\frac{PC_t G_t}{P_t Y_t} = gs_t + e_t^G$$

fluctuates systematically with the business cycle according to the following rule

(13b) $$gs_t = t_g^Y \log(Y_t / Ypot_t)$$

where t_g^Y measures the degree of automatic stabilisation of government expenditure and output gap is defined by:

(13c) $$Y_t / Ypot_t = ucap_t^{(1-\alpha)} \left(\frac{L_t}{L^{ss}} \right)^{\alpha}.$$

Discretionary fiscal action is characterised by the variable e_t^G which is allowed to be autocorrelated process
$e_t^G = (1-\rho^G) s^{GN} + \rho^G e_{t-1}^G + \varepsilon_t^G$, $\varepsilon_t^G \sim N(0, \sigma^G)$
where s^{GN} is the long run nominal government to GDP share.
Implicitly it is assumed that government expenditure is financed, by lump sum taxes.

Central bank policy rule (interest rate rule):
Monetary policy is modelled via the following Taylor rule, which allows for some smoothness of the interest rate response to the inflation and output gap

(14a)
$$\begin{aligned} inom_t = {} & ilag * inom_{t-1} + \\ & (1-ilag) * (Ex.R + g^U + \pi^T + t_M^{\pi} (\pi_{t-1}^C - \pi^T) + t_M^U \log(ucap_t) + t_M^L \log(L_t / L^*)) + \\ & t_M^{\Delta\pi} (\pi_t^C - \pi_{t-1}^C) + t_M^{\Delta U} \log(ucap_t / ucap_{t-1}) + t_M^{\Delta L} \log(L_t / L_{t-1}) + e_t^M \end{aligned}$$

The term e_t^M captures autocorrelated discretionary shocks to monetary policy
$e_t^M = \rho^M e_{t-1}^M + \varepsilon_t^M$, $\varepsilon_t^M \sim N(0, \sigma^M)$
and π^T is the inflation target. It is assumed that both fiscal and monetary authorities base their policies on a concept of output gap which is a function of ucap and L:

(15) $$Ygap_t^{inom} = ucap_t^{\alpha U} \left(\frac{L_t}{L^*} \right)^{\alpha L}$$

where the original constraint implied by the Cobb-Douglas production function ($\alpha U = 1 - \alpha L$) has been relaxed, by estimating separate coefficients for *ucap* and *L* terms.

1.4 Trade and the current account

So far we have only determined aggregate consumption, investment and government purchases but not the allocation of expenditure over domestic and foreign goods. In order to facilitate aggregation we assume that households, the government and the corporate sector have identical preferences across goods used for private consumption (*C*), public consumption (*G*) and investment (*I*). Let $X^l \in \{C^l, I^l, G\}$ be demand of an individual household, investor or the government, then her preferences are given by the following utility function

$$\text{(16a)} \quad X^l = \left[\omega^{\frac{1}{\sigma}} X^{l\,d\,\frac{\sigma-1}{\sigma}} + (1-\omega)^{\frac{1}{\sigma}} X^{l\,f\,\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{(\sigma-1)}}$$

where $X^{l\,d}$ and $X^{l\,f}$ are indexes of demand across the continuum of differentiated goods produced respectively in the domestic economy and abroad, given by.

$$\text{(16b)} \quad X^{l\,d} = \left[\sum_{h=1}^{n^d} \left(\frac{1}{n^d} \right)^{\frac{1}{\tau}} x_h^{l\,d\,\frac{\tau-1}{\tau}} \right]^{\frac{\tau}{\tau-1}} \quad , \quad X^{l\,f} = \left[\sum_{h=1}^{n^f} \left(\frac{1}{n^f} \right)^{\frac{1}{\tau}} x_h^{l\,f\,\frac{\tau-1}{\tau}} \right]^{\frac{\tau}{\tau-1}}$$

We define τ_t to measure the inverse of the time varying elasticity of demand for variety *h* and assume that it is itself stochastic and also allow for some cyclical variation. The term τ_t is given by (see eq. (12e'))

$$\text{(16c)} \quad \tau_t = \tau^0 + \tau^1 \log(Y_t / Ypot_t) + e_t^{\tau}$$

The elasticity of substitution between bundles of domestic and foreign goods $X^{l\,d}$ and $X^{l\,f}$ is σ. Thus aggregate imports are given by

$$\text{(17)} \quad IM_t = (1-\omega^X) \left(\frac{PC_t}{PM_t} \right)^{\sigma} (C_t + I_t + G_t) \exp(e_t^{IM}) \left(\frac{(IM_{t+1} / IM_t)^{sfim}}{(IM_{t-1} / IM_{t-2})^{(1-sfim)}} \right)^{\gamma_{IM} \cdot \beta}$$

where *PC* and *PM* is the (utility based) consumer price deflator and import price deflator respectively and e_t^{IM} is a random walk with drift, to allow for different trends in imports w.r.t. the common trend g^U in GDP, investment, consumption:

$$\text{(17a)} \quad \Delta e_t^{IM} = g^X + \rho^{IM} \Delta e_{t-1}^{IM} + \varepsilon_t^{IM}, \qquad \varepsilon_t^{IM} \sim N(0, \sigma^{IM}).$$

We assume similar demand behaviour in the rest of the world, therefore exports can be treated symmetrically and are given by

$$\text{(18)} \quad EX_t = (1-\omega^X) \left(\frac{(PC_t^W E_t)^{\alpha^X}}{P_t^{(1-\alpha^X)}} \frac{1}{PX_t} \right)^{\sigma^X} \frac{(Y_t^W)^{\alpha^X}}{Y_t^{(1-\alpha^X)}} \exp(e_t^{EX}) \left(\frac{(EX_{t+1} / EX_t)^{sfex}}{(EX_{t-1} / EX_{t-2})^{(1-sfex)}} \right)^{\gamma_{EX} \cdot \beta}$$

where PC_t^W are world prices (in foreign currency); PX_t and E_t are export prices an the nominal exchange rate and e_t^{EX} is a random walk with the same drift as e_t^{IM}:

(18a) $\Delta e_t^{EX} = g^X + \rho^{EX} \Delta e_{t-1}^{EX} + \varepsilon_t^{EX}$, $\qquad \varepsilon_t^{EX} \sim N(0, \sigma^{EX})$

Prices for exports and imports are set by domestic and foreign exporters respectively. The exporter buys goods from domestic producers and sells them in the foreign (domestic) market. It is assumed that the exporters are monopolistically competitive in their respective export markets. Exporters charge a mark-up over domestic prices (linear technology) and their pricing is subject to convex adjustment costs . Thus export prices are given by

(19a) $$\eta_t^{Xj} PX_t = cxP_t \left(\frac{P_t^C}{P_t} \right)^{1-sxd}$$

with a mark-up factor determined by

(19b) $$\log(\eta_t^{Xj}) = 1 - (\tau^{X0} + e_t^{\tau X}) + \gamma_{PX} \left[\beta(sfpx * \pi_{t+1}^{Xj} + (1 - sfpx)\pi_{t-1}^{Xj}) - \pi_t^{Xj} \right] \quad 0 \le sfpx \le 1$$

where $e_t^{\tau X}$ follows a random walk process: $\Delta e_t^{\tau X} = \varepsilon_t^{\tau X}$, $\qquad \varepsilon_t^{\tau X} \sim N(0, \sigma^{\tau X})$

We assume that monopolistic competition applies to the foreign firms as well. There is no pricing to market and import prices are given by

(20a) $$PM_t = cxw \left(\frac{(E_t PW_t)^{\alpha^X}}{P_t^{1-\alpha^X)}} \right)^{spm} (PM_{t-1})^{(1-spm)} \exp(e_t^{PM})$$

where e_t^{PM} follows a random walk process: $\Delta e_t^{PM} = \varepsilon_t^{PM}$, $\quad \varepsilon_t^{PM} \sim N(0, \sigma^{PM})$

Exports and imports together with interest receipts/payments determine the evolution of net foreign assets (*BW*)

(21) $$E_t BW_t = (1 + inom_t) E_t BW_{t-1} + PX_t EX_t - PM_t IM_t$$

Equivalently expressed in terms of stationary variables as:

(21a) $$BWRY_t = (1 + r_t)/(1 + GY_t) BWRY_{t-1} + NTBY_t$$

where $NTBY_t = \dfrac{PX_t EX_t - PM_t IM_t}{P_t Y_t}$, $BWRY_t = \dfrac{E_t BW_t}{P_t Y_t}$, GY is the GDP growth rate.

Finally, the interest parity condition is given by:

(22) $$inom_t = inomw_t + GE_{t+1} - rp^E BWRY_t + e_t^{RPE}$$

Where rp^E is the exchange rate risk premium, GE is the growth rate of the exchange rate and e_t^{RPE} is an autocorrelated shock to parity condition: $e_t^{RPE} = \rho^{RPE} e_{t-1}^{RPE} + \varepsilon_t^{RPE}$, with $\varepsilon_t^{RPE} \sim N(0, \sigma^{RPE})$.

1.5 The long run ratios implied by this model

The model implies a set of stationary long run ratios (some of them are conditional on the stationarity of the stochastic shocks). Whether these ratios are consistent with the data provides a first check for the model and its stochastic specification. These long run ratios relate to investment, consumption, the labour market and foreign trade. Denote the long run components with an asterisk, then these long run ratios can be represented as follows.

Investment:

Using the first order condition of the firm w. r. t. capital and investment together with the capital accumulation rule allows to express the nominal investment to GDP ratio as follows

$$\frac{PI^{*}I^{*}}{P^{*}Y^{*}} = \frac{(1-\alpha)\eta(g^{U}+\delta)}{g^{U}/\beta-1+\delta}$$

Possible trend variations of the nominal investment share must be matched with variations in the TFP trend.

Consumption:

The long run nominal private consumption share can be calculated by substituting the first order conditions for consumption into the intertemporal budget constraint. This yields the following expression

$$\frac{PC^{*}C^{*}}{P^{*}Y^{*}} = (1-\beta)\left[\frac{(1-\alpha)(1-\tau_{t})}{g^{U}/\beta-1+\delta} + \frac{\tau_{t}+\alpha(1-\tau_{t})}{g^{U}/\beta-1}\right]$$

Labour market:

From the firm's first order conditions for labour we obtain the following expression of the wage share

$$\frac{W^{*}L^{*}}{P^{*}Y^{*}} = \frac{\alpha\eta^{*}}{1-lol^{*}}\text{, where } lol \equiv \frac{LO}{L}$$

In order to match the model specification with the observed declining trend in the wage share one either has to allow for a trend decline in η (trend increase in the mark-up) or a trend decline in overhead labour (lol).

From the labour supply rule we obtain a relationship between the employment rate and the wage share where we make use of the ratio between nominal consumption and GDP

$$\omega(1-L^{*}) = \left[\frac{\theta-1}{\theta} const \frac{W^{*}L^{*}}{P^{*}Y^{*}}\right]$$

Stationarity of the employment rate depends on the absence of a trend in the wage share and stationarity of the leisure shock.

Foreign trade:
Under the general assumption that domestic and foreign real interest rate are not equal in the steady state, the trade balance:
$BWRY^{*} = (inomw^{*} - inom^{*}) / rp^{E}$
And
$NTBY^{*} = (-Ex.R) / (1 + g^{U}) \cdot BWRY^{*}$

2. Estimation

2.1 Priors

In Table 1 we show the information about prior distributions of standard errors of shocks, AR coefficients of autocorrelated shocks and structural parameters. The following parameters as kept constant over the estimation exercise:

$\alpha = 0.52$

$\tau = 0.1$

$a_1 = (1-\tau)*(1-\alpha)1/\text{KSN}$, determined in order to assure the steady state constraint UCAP=1, where $KSN = K/Y*PC/P$ is the nominal capital to GDP share.

ω is determined in order to assure the steady state condition $L^* = 0.62$

rp is determined in order to assure the steady state condition $ISN = 0.21$

Finally, ρ^{LOL} is fixed at 0.99 in order to estimate a persistent and 'smooth' pattern in LOL.

2.2 Mapping of the stability region of model coefficients

When the degree of complexity of the model structure and its parameterization increases, it might not be trivial to know a priory the set of model coefficients assuring the rank condition for the solution of forward looking components (Blanchard and Kahn, 1980). In this case, some simple mapping procedures can be applied based on the filtering of Monte Carlo samples of model coefficients within prior bounds.

This procedure is as follows:

1) a Monte Carlo sample of model coefficients is produced sampling from uniform distributions within prior bounds;
2) for each element of the sample, the model is linearised and the rank condition is checked;
3) the original MC sample is filtered according to the compliance to the rank condition, obtaining to subsamples.
   B (behavior) for successful parameter sets;
   $\overline{B}$ (non-behavior) for unsuccessful sets;
4) the Smirnov test is applied to compare the prior sample with, e.g., the filtered $\overline{B}$ sample, to identify which of the model coefficients mostly drive the model to violate the rank condition; this can also lead to a revision of hypothesized prior bounds;
5) further inspection of bi-dimensional projections of the $\overline{B}$ and B samples onto planes can further provide additional information about bad combinations of model coefficients.

We show below an example for the model under analysis. The most relevant parameters for rank condition were γ_{EX}, γ_{IM}, *sfex*, *sfim*, *ilag*, *sfp*.

Figure 1 shows the analysis of the marginal distributions for the six relevant coefficients under the $\overline{B}$ subset. In the upper panels we can see that high values of γ_{EX}, γ_{IM}, *ilag* and low values of *sfex*, *sfim* and *sfp* mostly drive the rank condition to be violated. In Figure 2 we show the comparison between the cumulative distributions of the original sample (uniform distributions; diagonal lines) and the filtered sample $\overline{B}$.
In Figure 3 we see a further inspection of the B sample by projecting it onto the γ_{EX}-*sfex* and γ_{IM}-*sfim* planes. The dots represent the parameter combinations that fulfil the rank condition and, conversely, the white region is the region violating the rank condition. We can see that the coefficients violating the rank conditions are concentrated in a triangular region in the lower-right part of the planes.

2.3 Estimation results

The estimation was performed applying the same approach as Schorfheide (2000), Smets and Wouters (2003). From the computational point of view, the DYNARE toolbox for MATLAB has been applied (Juillard, 1996-2005).
In Table 2 we show summary statistics of the simulated posterior distributions of the estimated parameters; we show the prior and posterior plots for standard errors of shocks in Figure 4, of AR coefficients in Figure 5 and of structural parameters in Figures 6-9.
In Figures 10-11 we show the 1-step ahead predictions of the DSGE model[3], while in Figures 12-14 we show the posterior mean and Highest Posterior Density interval of smoothed shocks. In Figures 15-17 we show the same for unobserved variables.

The draws from the posterior distribution has been obtained via Markov Chain Monte Carlo, computing 4 parallel chains, each of 10,000 draws, and using the last 50% of draws for the analysis. Convergence of the chains has been assessed by the Brooks and Gelman (1998) criteria.

The information set is given by: GC, GE, GEX, GI, GIM, GL, GY, π^W, inom, π, π^C, π^M, π^X, inomw, π^{Wld}, GYW. World economy series [inomw, π^{Wld}, GYW] are considered as exogenous and are modeled with a VAR(1) process. Data range from 1980-1Q to 2003-4Q for EURO zone.

3. Testing

3.1 Simple comparisons of DSGE with a VECM

During the model building, simple comparisons of the DSGE model fit w.r.t. VECM or VAR models are performed, applying RMSE's. Even without the complete Bayesian model comparison, such tests can give an idea of critical parts of the DSGE model to be improved.
The VECM is defined as a VAR(1) in the observed variables, plus equilibrium corrections:

$\log(CSN) = \log(C/Y*PC/P)$ (nominal consumption to GDP share)

$\log(ISN) = \log(I/Y*PC/P)$ (nominal investment to GDP share)

[3] Growth rate variables have the 'G' prefix: e.g. GC is growth rate of consumption.

$\log(EX / Y * PX / P) - \log(IM / Y * PM / P)$ (nominal trade balance)
$\log(L)$
$\log(W / PY)$ (nominal wages to nominal GDP share)
We also allow the foreign variables as explanatory variables in the VAR.

A simple comparison between RMSE's from the DSGE model and those from the VECM show that the VECM generally outperforms the DSGE model (Table 3). However, the differences for the four step ahead prediction errors are much smaller than those for the one step ahead prediction errors. In fact, for many variables (consumption, exports, exchange rate) the DSGE errors are very close to those of the VECM for the four step ahead prediction errors. The comparison is worst for the wage inflation variable π^W, with one step ahead prediction errors more than 50% larger than that of the VECMs, and for nominal interest rate with four step ahead error more than double that of the VECM.

3.2 Diagnostics for the fit behaviour of the DSGE
In order to better understand the mechanisms that drive the fit of the DSGE model, we performed a detailed sensitivity analysis of the DSGE mean squared errors (MSE) of each observed series, versus model coefficients. We applied a global sensitivity analysis approach, whereby the MSE's are analysed by varying all coefficients simultaneously.
Therefore, we performed a Monte Carlo analysis by sampling the coefficients independently, with uniform distributions in the range defined by the *posterior* distribution. This option allows a clearer interpretation of results, since in this way sensitivity analysis results are not affected by the dependency structure of the full joint posterior distribution.

Also in this case, we applied a Monte Carlo filtering procedure. Starting from a sample of 1,200 runs, we apply a filtering rule by selecting the 10% runs with smallest MSE for each observed series. We then applied the Smirnov test to compare the original samples to the filtered one in order to detect the coefficients mainly driving the fit of each observed series. The critical level of significance was 0.002 %.

This analysis allows first to detect three subsets of coefficients:

1) Coefficients that significantly drive the fit of more than 1 series: these are shown in Figures 20-24.

	Observed series	notes
a_2	GC, GI, GIM, π, π^C, π^M, π^X	
γ_{EX}	GEX, GL	
γ_K	GC, GI, GY, π^C, π^M, π^X	
γ_I	GC, GE, GEX	
γ_P	GIM, π^C, π^M	
γ_{PX}	GEX, GIM, *inom*, π^X	
γ_W	π, π^W	(no trade off)
habl	GC, GL, GY, π^W, π	

ilag	GC, GI, GY, inom, π^C, π^M, π^W	
κ	GEX, GL	
ρ^C	GC, GY, inom	
ρ^{EX}	GE; GEX, GY	
ρ^{IM}	GE, GIM	(no trade off)
ρ^L	GL, π^W, inom, π, π^C, π^M, π^X	
ρ^M	GL, GY, inom	
ρ^{RPE}	GC, GE, inom, π^M	
ρ^{rp}	GL, GY, inom	(no trade off)
ρ^U	GI, GY	(no trade off)
RPE	GE, π^C	
sfex	GEX, π	
sfp	π, π^C	(no trade off)
sfpx	inom, π^X	(no trade off)
spm	GE, GEX, GIM	(no trade off)
σ	GE, GIM	(no trade off)
θ	π, π^C, π^X, π^W	(no trade off)
$t_M^{\Delta\pi}$	GI, π^M	(no trade off)
$t_M^{\Delta L}$	GC, GE, GY, inom	
t_M^L	GC, GE, GL	
t_M^U	GE; GI, GL, GY, π^M	

Figures 20 to 24 show the trade offs in the fit of endogenous variables to the choice of structural parameters. Here we discuss two parameters, γ_{PX} and *ilag*. In the case of γ_{PX} one can see a conflict between the fit of exports and the fit of export prices. In order to fit export prices a high value of the smoothness parameter is required, while a good fit of exports would require a low smoothness parameter. In other words shocks affecting domestic prices should be transmitted more quickly into export prices in order to obtain a good fit for exports. This could point to a misspecification of the export price equation. There is probably an unobserved stochastic productivity growth differential that explains part of the growth differential between the export and GDP deflator which is not taken into account in our specification. This specification error is partly captured in the lagged growth rate of export prices (if the missing variable is itself autocorrelated) and therefore leads to an overestimation of price rigidity.

A similar phenomenon occurs with the *ilag* parameter. Here again a good fit of the nominal interest rate requires a high parameter value for *ilag* while a good fit of the growth rate of consumption, investment and GDP requires a small value for *ilag*. Measurement error of the variables entering the Taylor rule could be a possible explanation. Notice, there are two unobserved variables, the inflation target and the output gap. Both variables are most likely estimated with error and both of them are likely to be autocorrelated. This could bias the lagged interest rate in the Taylor rue in an upward direction.

2) Coefficients that significantly drive the fit of only one series;

	Observed series
α^X	GEX
g^X	GEX
habc	GC
ρ^τ	π^C
ω^X	GC
sfw	π^W
σ^X	GEX
$t_M^{\Delta U}$	inom
t_M^π	π^M

3) Less-influent coefficients,

s^{CN}
t_g^Y
γ_L
γ_{IM}
ρ^G
ρ^I
ρ^{PC}
sfim
sxd
τ^1

Such coefficients are not necessarily non-influent. They mostly drive the fit through the correlation and interaction structure characterising the posterior joint distribution.

4. Sensitivity analysis for IRFs

When the posterior distribution of the model coefficients has been estimated for the final DSGE model, global sensitivity analysis techniques can be used to map the uncertainty distribution of model results onto the model coefficients.
The quantiles of the IRF distributions obtained with the Bayesian procedure are shown in Figures 38-56. Hence, Bayesian analysis provides a full uncertainty distribution of IRF's. From this, it could be of interest to identify which parameters mostly drive the uncertainty distribution of, say, the IRF to a technological shock or the IRF to a monetary shock.

Among the very large number of input output tables in the IRF analysis, we selected a few of them, that present a wider uncertainty range, and for which an in depth sensitivity analysis can be of interest.

IRF's that seem to have quite large uncertainty bounds are those for unit shocks in ε^L. We therefore considered the sensitivity of the responses of the following variables, selecting the time point where the uncertainty seems to be the largest:

- GC, GI, GY, π, π^C, π^M, π^X at t=1;
- L at t=12;
- inom at t=6.

The sensitivity analysis procedure, in this case, is based on computing standardised regression coefficients (SRC) of the given IRF vs. the set of model coefficients.

SRC provide a regression model in terms of standardised variables

$$\tilde{y}(t) = \frac{y(t) - \bar{y}(t)}{\sigma_y(t)}; \quad \tilde{p}_x = \frac{p_x - \bar{p}_x}{\sigma_x}$$

$$\hat{y}(t) = \sum_{x=1\ldots n} \hat{\beta}_x \tilde{p}_x$$

Where *y(t)* is the generic response at time *t* and $\hat{y}(t)$ is the regression model prediction. The SRC's $\hat{\beta}$ are normalised in the interval [-1, 1] and their squared value represent the fraction of the variance of y that is explained by a linear function of each model parameter p_x (see Saltelli et al., 2004, for a review on global sensitivity analysis measures).

For our case study ε^L we get the following results (relevant SRC's are reported in the tables below).

t=1:

	GC	GI	GY	π	π^C	π^M	π^X
γ^I		-0.139					
γ^W	-0.122			-0.151	-0.149	-0.128	-0.131
habc	-0.206						
ilag	0.298	0.299	0.297	0.119	0.138	0.204	0.123
κ	-0.265	-0.189	-0.245	-0.406	-0.400	-0.347	-0.417
ρ^L		0.445	0.356	0.497	0.496	0.460	0.548
sfp	-0.154		-0.156	0.232	0.185		
sfpx							0.102
spm						0.122	
$t_M^{\Delta\pi}$	-0.103						
$t_M^{\Delta L}$	0.566	0.525	0.532	0.208	0.243	0.366	0.206
θ	0.124	0.002	0.067	0.180	0.173	0.135	0.159
t_M^{π}	-0.111	-0.138	-0.133	-0.097	-0.101	-0.109	-0.099
t_M^{L}	0.4083	0.3405	0.4191	0.3944	0.4092	0.4387	0.432

L at t=12;

γ^W	0.1288
κ	0.5928
ρ^L	-0.654
θ	-0.2216

inom at t=6.

κ	-0.3498
ρ^L	0.6245
t_M^L	0.5035

κ, ρ^L, t_M^L and $t_M^{\Delta L}$ have the highest SRC's, that can be as high as 0.5, i.e. accounting, alone, for large part of the whole uncertainty bound of the response (SRC=0.5 corresponds to a 25% contribution to the overall uncertainty).

5 Conclusions

We have estimated an open economy model for the EURO area that reasonably fits the actual data, in which we had a careful analysis of trends that allowed for a stochastic trend specification in technology. Different trends have also been considered, such as in import/exports w.r.t. GDP, consumption and investment.
Few transition phenomena towards a steady state have to be faced, as well:
a) the decline of wage share over time is interpreted with declining labour overhead;
b) falling in inflation rate is a more problematic issue, still open, above all in the Taylor rule specification;
c) interpretation of the declining technology in the final part of the observation period is still lacking.

The simple comparison with a VECM specification, shows that the VECM outperforms the DSGE as far as 1-step ahead predictions are concerned, while 4-step are much more similar. This can be at least partially explained by a residual seasonality in the data.
We applied global sensitivity analysis tools to analyse more in-depth possible mis-specification problems in the Taylor rule and in the description of trade variables.
Possible extensions can be then outlined from the estimation performed:
1) two different technology trends can be introduced for tradable and non-tradable sectors (in relation to different growth rates in export and import prices relative to GDP deflator)
2) in the Taylor rule we still are lacking a satisfactory model for the output gap. We have currently tried using a specification based on capacity utilisation and labour (looks that L gap has a bigger weight than ucap gap), and still lacked a reasonable model for the change in inflation target over time, in such a way that we currently preferred to set a constant target. The high estimated *ilag* coefficient seems an indication of that.

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Table 1

	distribution	Mean	St. dev	Support
Shocks				
σ^{C}	Beta	0.1	0.04	[0 0.2]
σ^{τ}	Beta	0.1	0.04	[0 0.2]
$\sigma^{\tau X}$	Beta	0.1	0.04	[0 0.2]
σ^{EX}	Beta	0.1	0.04	[0 0.2]
σ^{IM}	Beta	0.1	0.04	[0 0.2]
σ^{G}	Beta	0.1	0.04	[0 0.2]
σ^{I}	Beta	0.01	0.004	[0 0.02]
σ^{L}	Beta	0.1	0.04	[0 0.2]
σ^{LOL}	Beta	0.1	0.04	[0 0.2]
σ^{M}	Beta	0.1	0.04	[0 0.2]
σ^{PC}	Beta	0.1	0.04	[0 0.2]
σ^{PM}	Beta	0.1	0.04	[0 0.2]
σ^{rp}	Beta	0.1	0.04	[0 0.2]
σ^{RPE}	Beta	0.1	0.04	[0 0.2]
σ^{U}	Beta	0.1	0.04	[0 0.2]
σ^{W}	Beta	0.02	0.008	[0 0.04]
AR coefficients				
ρ^{τ}	Beta	0.5	0.2	[0 1]
ρ^{C}	Beta	0.5	0.2	[0 1]
ρ^{EX}	Beta	0.5	0.2	[0 1]
ρ^{G}	Beta	0.5	0.2	[0 1]
ρ^{I}	Beta	0.5	0.2	[0 1]
ρ^{IM}	Beta	0.5	0.2	[0 1]
ρ^{L}	Beta	0.5	0.2	[0 1]
ρ^{M}	Beta	0.5	0.2	[0 1]
ρ^{PC}	Beta	0.5	0.2	[0 1]
ρ^{RPE}	Beta	0.5	0.2	[0 1]
ρ^{rp}	Beta	0.5	0.2	[0 1]
ρ^{U}	Beta	0.5	0.2	[0 1]
Structural parameters				
a_2	Beta	0.1	0.04	[0 0.2]
α^{X}	Beta	0.5	0.2	[0 1]
g^{X}	Beta	0.007	0.0025	[0 0.02]
s^{CN}: long run nominal	Beta	0.575	0.006	[0.56 0.59]

consumption to GDP share				
t_g^Y	Beta	0	0.4	[-1 1]
γ_{EX}	Beta	0.5	0.2	[0 1]
γ_{IM}	Beta	0.5	0.2	[0 1]
γ_K	Beta	15	6	[0 30]
γ_I	Beta	15	6	[0 30]
γ_L	Beta	15	6	[0 30]
γ_P	Beta	15	6	[0 30]
γ_{PX}	Beta	15	6	[0 30]
γ_W	Beta	15	6	[0 30]
habc	Beta	0.45	0.18	[0 0.9]
habl	Beta	0.45	0.18	[0 0.9]
ilag	Beta	0.8	0.1	[0 1]
κ	Gamma	0.5	0.4	[0 ∞]
rp^E	Beta	0.01	0.004	[0 0.02]
ω^X	beta	0.8	0.04	[0.7 0.9]
sfex	Beta	0.5	0.2	[0 1]
sfim	Beta	0.5	0.2	[0 1]
sfp	Beta	0.5	0.2	[0 1]
sfpx	Beta	0.5	0.2	[0 1]
sfw	Beta	0.5	0.2	[0 1]
spm	Beta	0.6	0.16	[0.2 1]
sxd	Beta	0.5	0.2	[0 1]
σ	Beta	1.25	0.3	[0.5 2]
σ^X	Beta	1.25	0.3	[0.5 2]
τ^1	Beta	-0.1	0.03	[-0.2 0]
θ	Gamma	2	0.8	[1 ∞]
$t_M^{\Delta\pi}$	Beta	0.2	0.08	[0 0.4]
$t_M^{\Delta L}$	Beta	1	0.4	[0 2]
$t_M^{\Delta U}$	Beta	0.1	0.04	[0 0.2]
t_M^{π}	Beta	1.25	0.1	[1 1.5]
t_M^{L}	Beta	0.75	0.3	[0 1.5]
t_M^{U}	Beta	0.75	0.3	[0 1.5]

Table 2. Posterior distributions

Shocks	post. mode	post. mean	HP interval inf	HP interval sup
σ^{C}	0.0431	0.0397	0.0302	0.0482
σ^{τ}	0.0396	0.0370	0.0262	0.0488
$\sigma^{\tau X}$	0.0165	0.0204	0.0138	0.0266
σ^{EX}	0.0215	0.0229	0.0194	0.0267
σ^{IM}	0.0283	0.0292	0.0250	0.0337
σ^{G}	0.0095	0.0098	0.0084	0.0112
σ^{I}	0.0100	0.0098	0.0035	0.0167
σ^{L}	0.0452	0.0541	0.0309	0.0795
σ^{LOL}	0.0077	0.0081	0.0060	0.0100
σ^{M}	0.0015	0.0016	0.0013	0.0018
σ^{PC}	0.0014	0.0014	8.9440e-004	0.0020
σ^{PM}	0.0120	0.0122	0.0109	0.0135
σ^{rp}	0.0013	0.0015	0.0010	0.0019
σ^{RPE}	0.0291	0.0291	0.0174	0.0417
σ^{U}	0.0076	0.0077	0.0064	0.0092
σ^{W}	0.0078	0.0086	0.0031	0.0142
AR coefficients				
ρ^{τ}	0.1316	0.1774	0.0257	0.3501
ρ^{C}	0.3344	0.3696	0.2080	0.5373
ρ^{EX}	0.0843	0.0885	0.0222	0.1540
ρ^{G}	0.8795	0.8832	0.8167	0.9447
ρ^{I}	0.5009	0.4820	0.1416	0.8114
ρ^{IM}	0.2811	0.2586	0.1687	0.3505
ρ^{L}	0.9533	0.9352	0.8903	0.9821
ρ^{M}	0.6617	0.6832	0.5728	0.7851
ρ^{PC}	0.1389	0.1558	0.0494	0.2625
ρ^{RPE}	0.9848	0.9726	0.9483	0.9960
ρ^{rp}	0.4549	0.4534	0.3264	0.5697
ρ^{U}	0.2744	0.3074	0.1148	0.4931
Structural parameters				
a_2	0.0068	0.0081	0.0039	0.0126
α^{X}	0.8515	0.8206	0.7235	0.9502
g^{X}	0.0048	0.0048	0.0022	0.0069
s^{CN}	0.5724	0.5737	0.5643	0.5825

t_g^Y	-0.6899	-0.6753	-0.9272	-0.4924
γ_{EX}	0.5265	0.4640	0.0998	0.7852
γ_{IM}	0.1473	0.1882	0.0329	0.3427
γ_K	14.3102	15.4774	7.7861	24.4355
γ_I	25.9601	24.7262	20.8590	29.1538
γ_L	27.6065	26.6840	24.0857	29.5377
γ_P	25.4605	24.0230	18.6769	29.2222
γ_{PX}	5.1080	7.6814	2.8040	12.5128
γ_W	8.1837	13.7743	4.3588	22.8603
habc	0.8759	0.8598	0.8301	0.8933
habl	0.7803	0.7538	0.6360	0.8850
ilag	0.7286	0.7045	0.6391	0.7714
κ	2.7020	2.9308	1.7922	3.9787
rp^E	0.0157	0.0152	0.0115	0.0190
ω^X	0.8415	0.8413	0.8226	0.8582
sfex	0.8682	0.7966	0.6414	0.9721
sfim	0.3287	0.4084	0.1244	0.6691
sfp	0.7103	0.7167	0.5844	0.8622
sfpx	0.9078	0.8848	0.7799	0.9769
sfw	0.9557	0.9359	0.8796	0.9923
spm	0.3823	0.3856	0.3240	0.4547
sxd	0.1373	0.2136	0.0408	0.3875
σ	1.6389	1.6408	1.4348	1.8735
σ^X	0.6398	0.7332	0.5530	0.9179
τ^1	-0.0900	-0.0920	-0.1383	-0.0461
θ	2.0102	2.4687	1.4037	3.5437
$t_M^{\Delta\pi}$	0.2820	0.2882	0.2316	0.3524
$t_M^{\Delta L}$	0.3845	0.4081	0.2096	0.6127
$t_M^{\Delta U}$	0.0479	0.0456	0.0215	0.0672
t_M^{π}	1.4392	1.4286	1.3660	1.4936
t_M^{L}	0.0797	0.1078	0.0165	0.1859
t_M^{U}	0.0685	0.0802	0.0440	0.1125

Table 3 Comparison of fit between DSGE and VECM

1-step ahead predictions

	RMSE DSGE	RMSE VECM
GC	0.005645	0.004155
GE	0.032403	0.025307
GEX	0.019222	0.015234
GI	0.014082	0.010394
GIM	0.015473	0.011921
GL	0.00122	0.001046
GY	0.005903	0.004382
π^W	0.007345	0.00478
inom	0.001415	0.001016
π	0.002678	0.001942
π^C	0.002755	0.002025
π^M	0.013443	0.011344
π^X	0.006494	0.005352
inomw	0.002588	0.002588
π^{Wld}	0.003571	0.003571
GYW	0.00426	0.00426

4-step ahead predictions

	RMSE DSGE	RMSE VECM
GC	0.004859	0.004649
GE	0.031425	0.029222
GEX	0.017665	0.016158
GI	0.013636	0.011981
GIM	0.013925	0.012585
GL	0.001996	0.001602
GY	0.005357	0.004971
π^W	0.0073	0.005784
inom	0.004756	0.002217
π	0.003343	0.002622
π^C	0.003538	0.002968
π^M	0.016907	0.015293
π^X	0.008698	0.007618
inomw	0.005012	0.005012
π^{Wld}	0.003319	0.003319
GYW	0.004758	0.004758

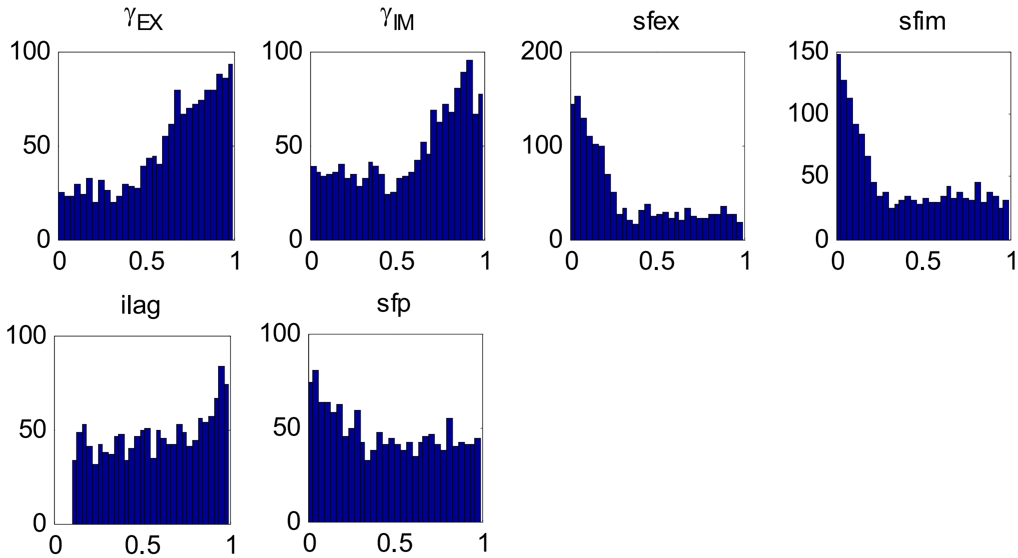

Figure 1

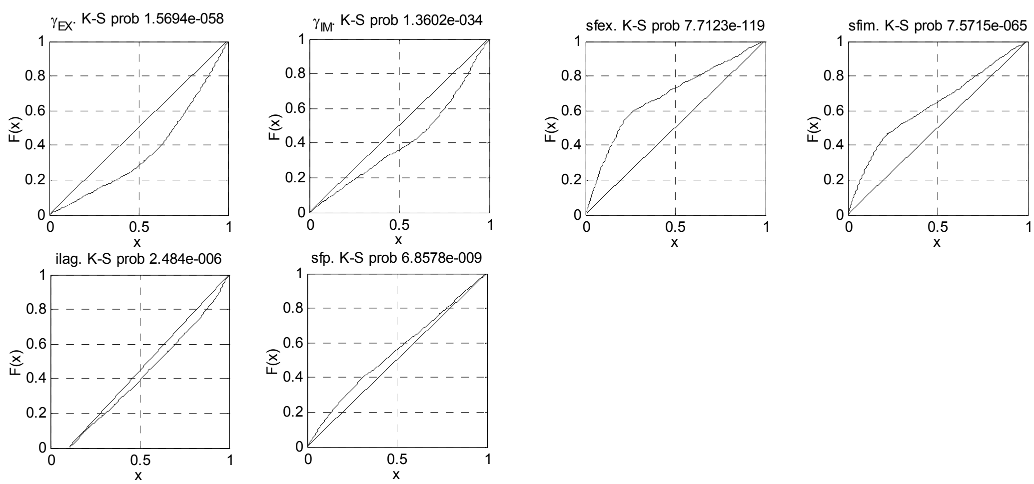

Figure 2

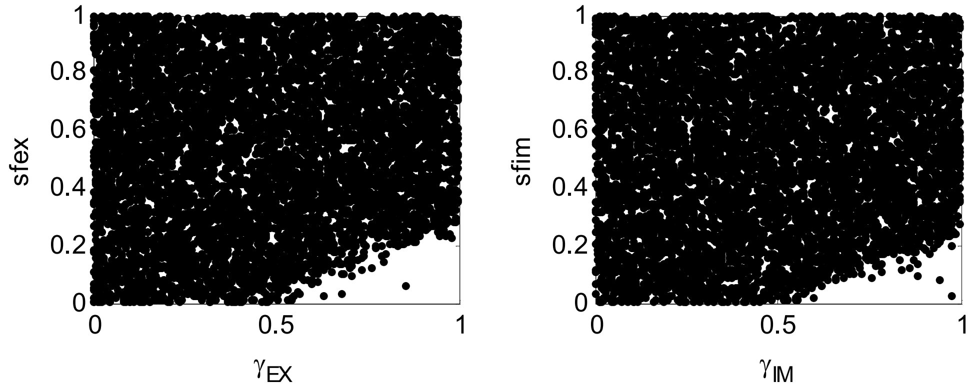
1
0.8
0.6
0.4
0.2
0
sfex
0
0.5
1
γ_{EX}
1
0.8
0.6
0.4
0.2
0
sfim
0
0.5
1
γ_{IM}

Figure 3

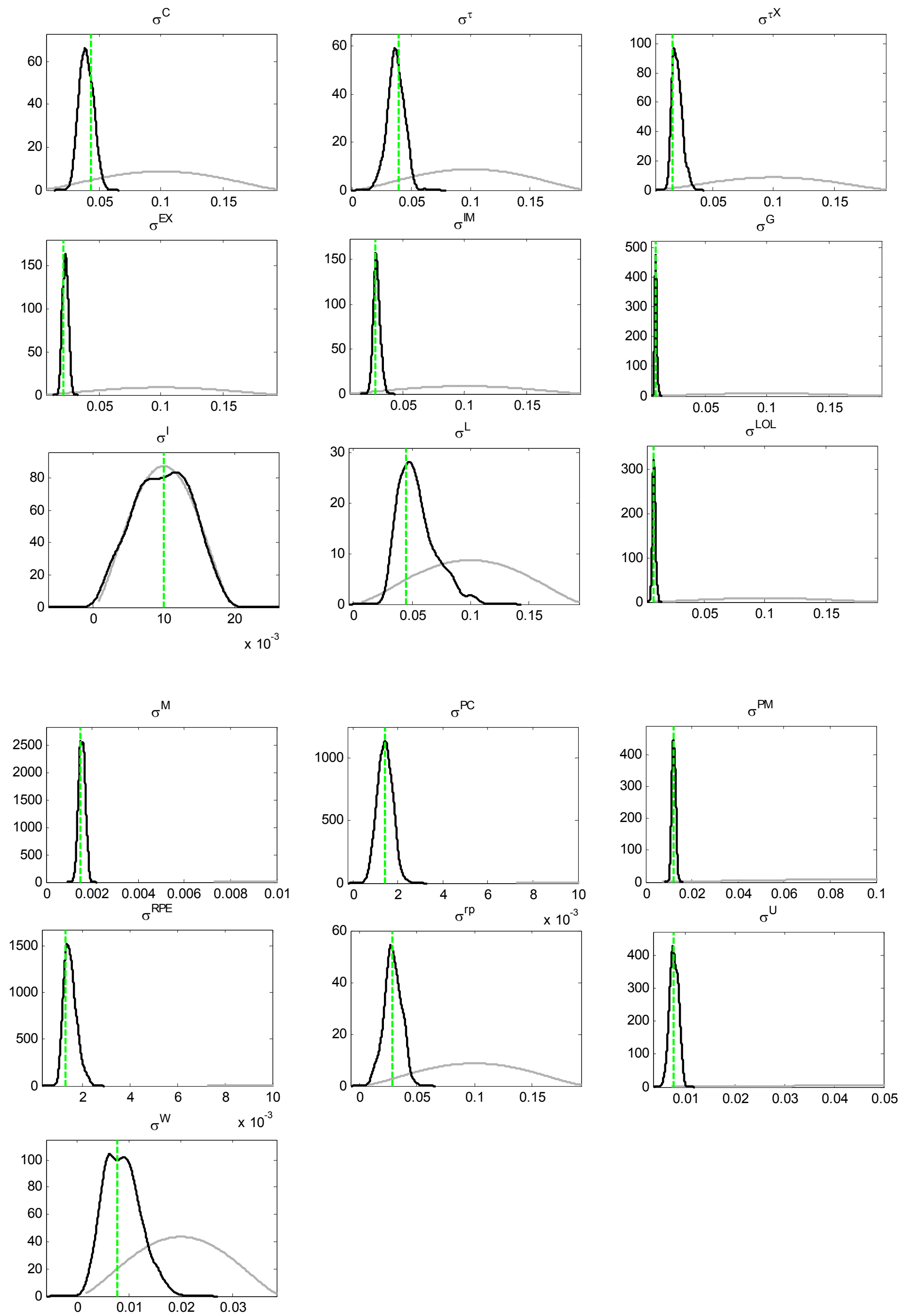

Figure 4. Priors (grey lines), posteriors (black lines) and posterior mode (vertical lines) of the estimated parameters.

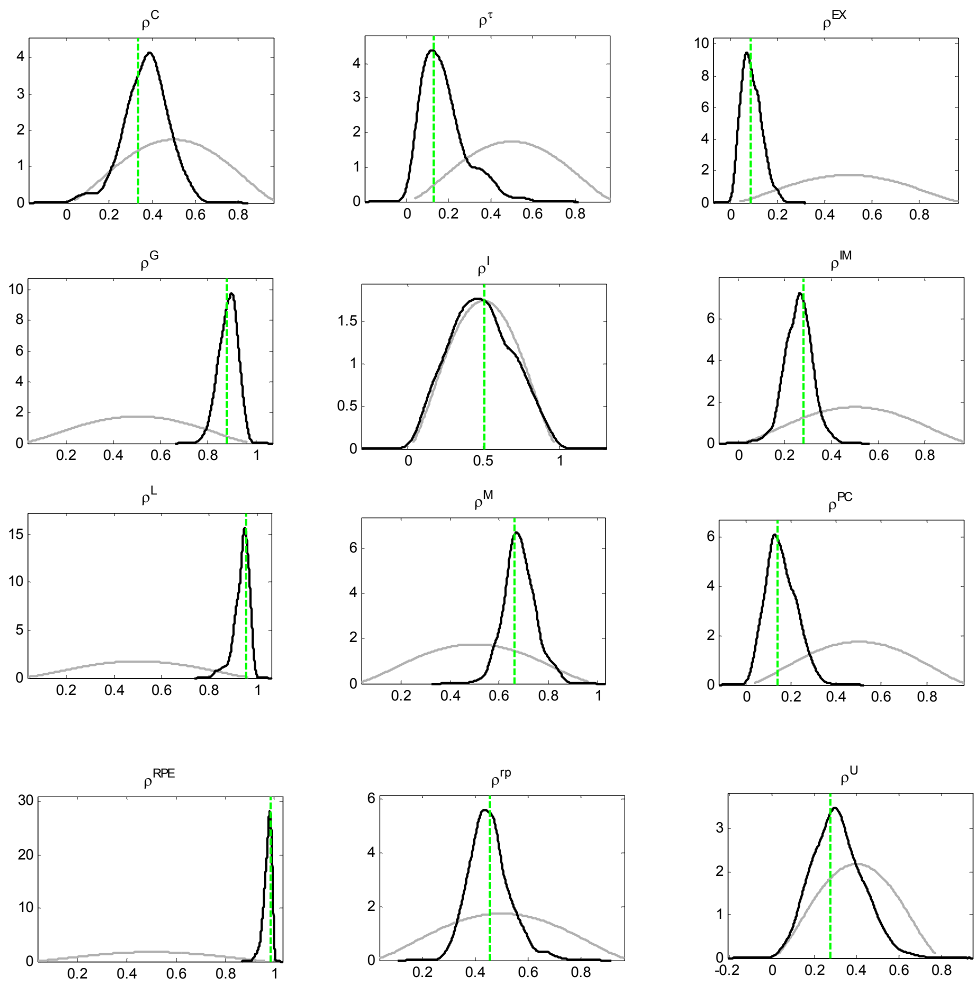

Figure 5. Priors (grey lines), posteriors (black lines) and posterior mode (vertical lines) of the estimated parameters.

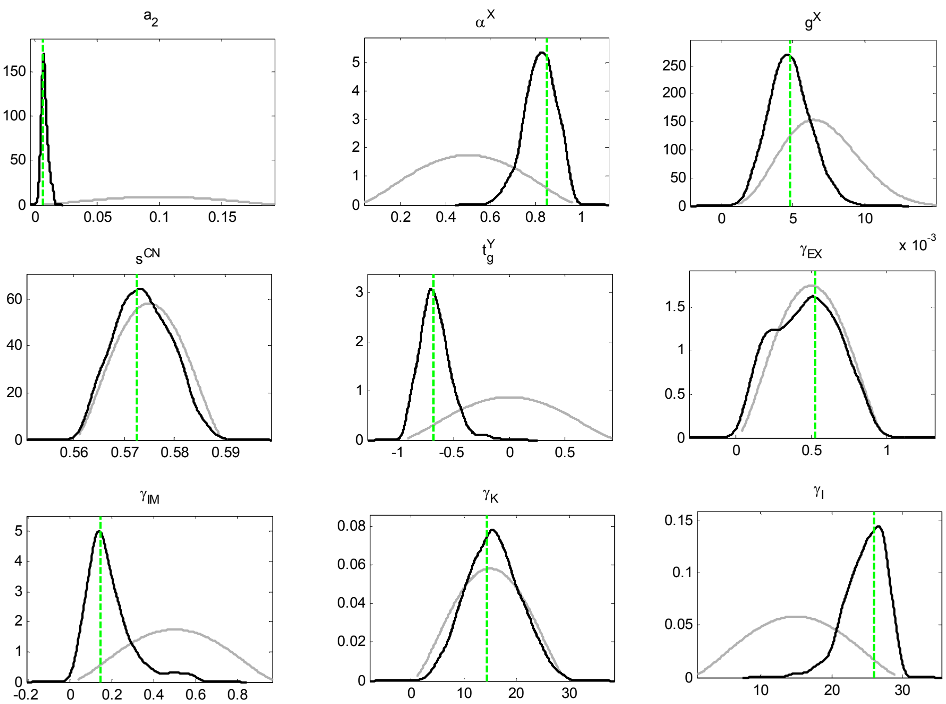

Figure 6. Priors (grey lines), posteriors (black lines) and posterior mode (vertical lines) of the estimated parameters.

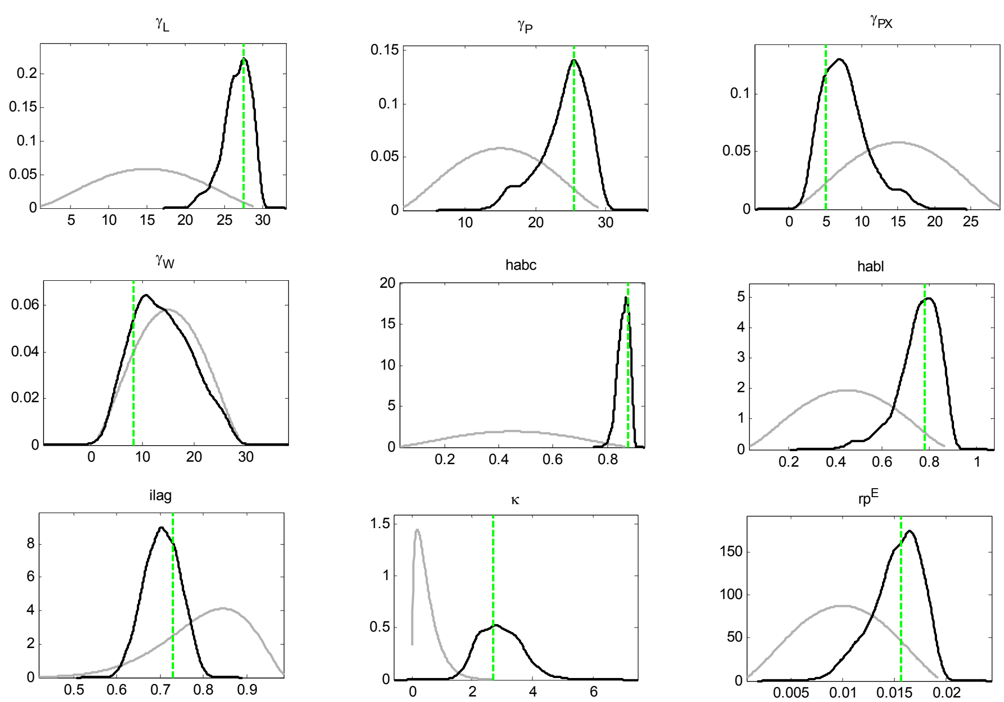

Figure 7. Priors (grey lines), posteriors (black lines) and posterior mode (vertical lines) of the estimated parameters.

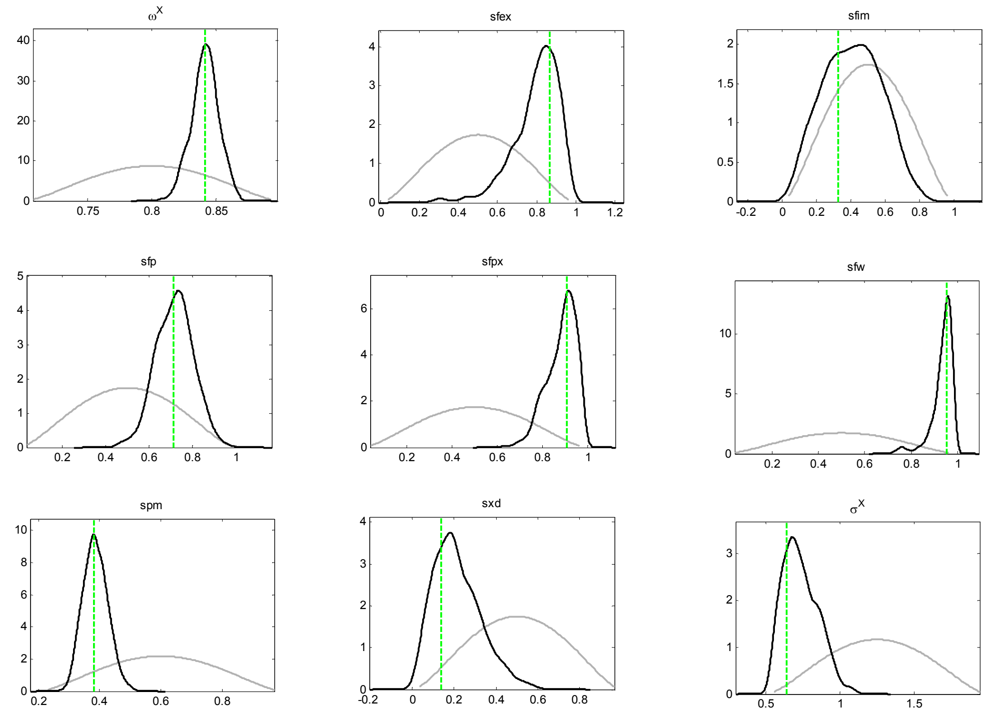

Figure 8. Priors (grey lines), posteriors (black lines) and posterior mode (vertical lines) of the estimated parameters.

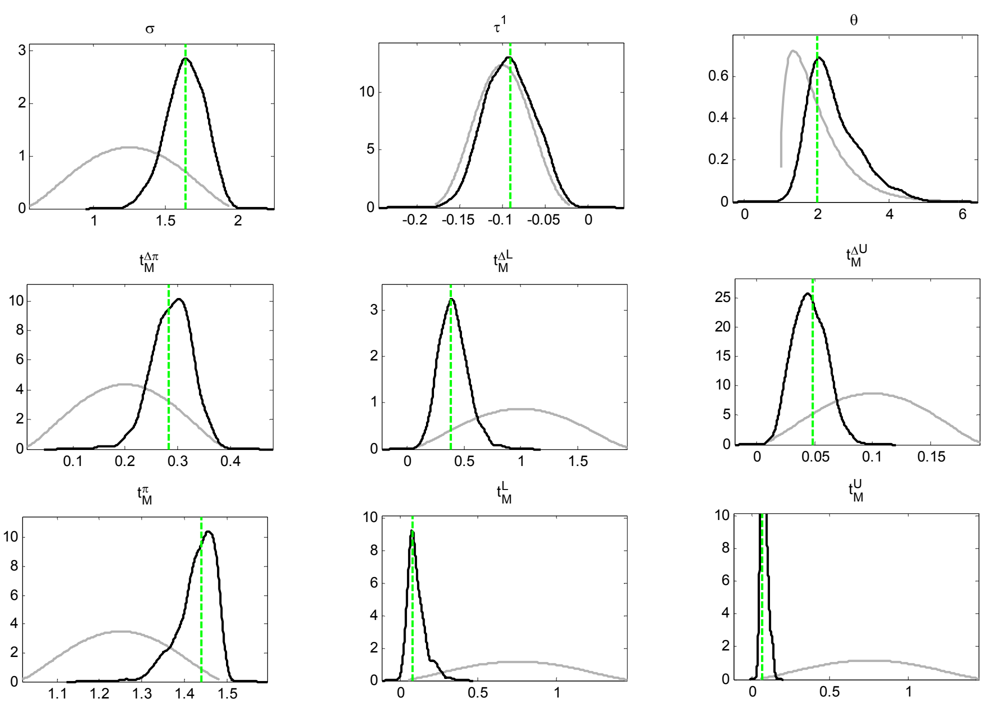

Figure 9. Priors (grey lines), posteriors (black lines) and posterior mode (vertical lines) of the estimated parameters.

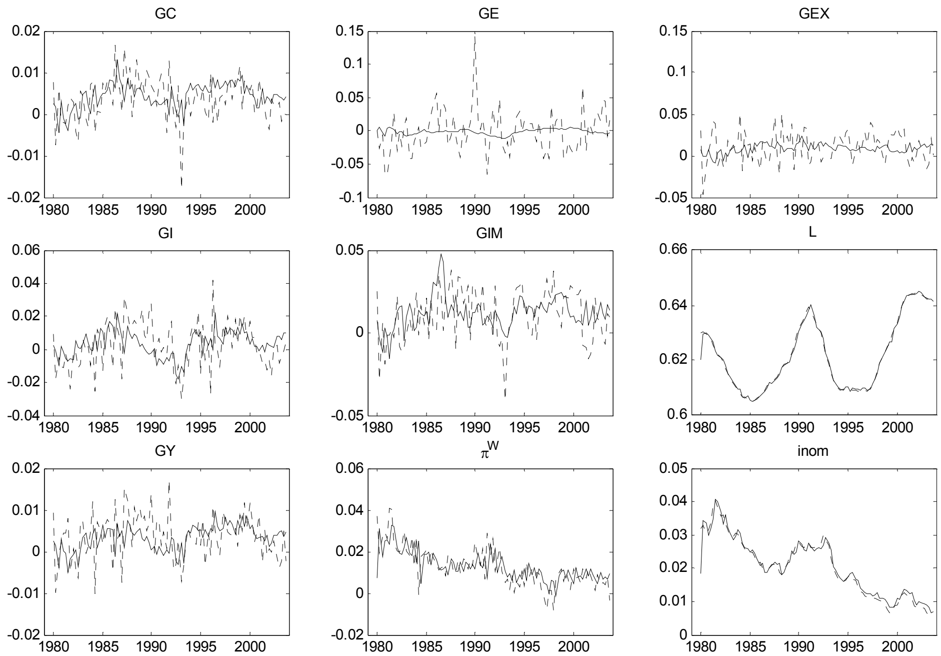

Figure 10. Fit of DSGE model: Solid lines are the posterior mean of one step ahead predictions; dashed lines are the data[1].

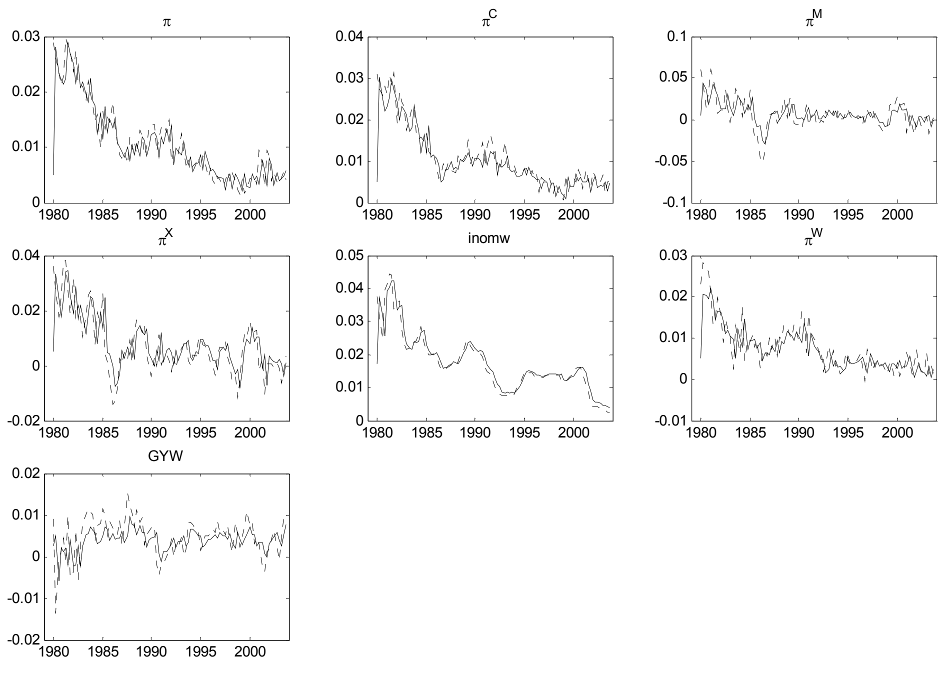

Figure 11. Fit of DSGE model: Solid lines are the posterior mean of one step ahead predictions; dashed lines are the data.

[1] growth rate variables have the 'G' prefix: e.g. GC is growth rate of consumption.

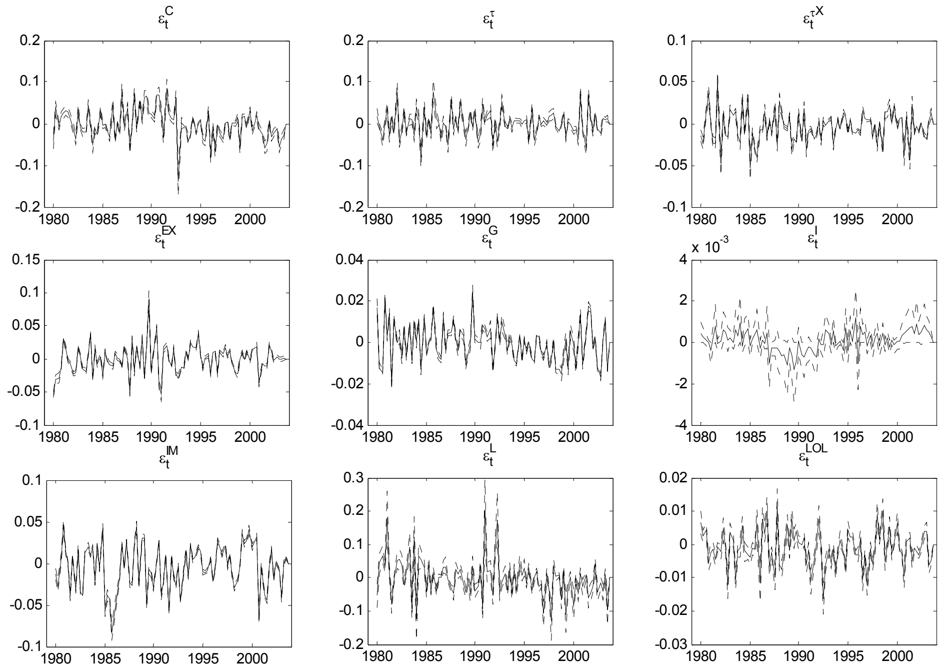

Figure 12. Posterior mean (solid) and Highest Posterior Densitiy interval (dashed) of smoothed shocks.

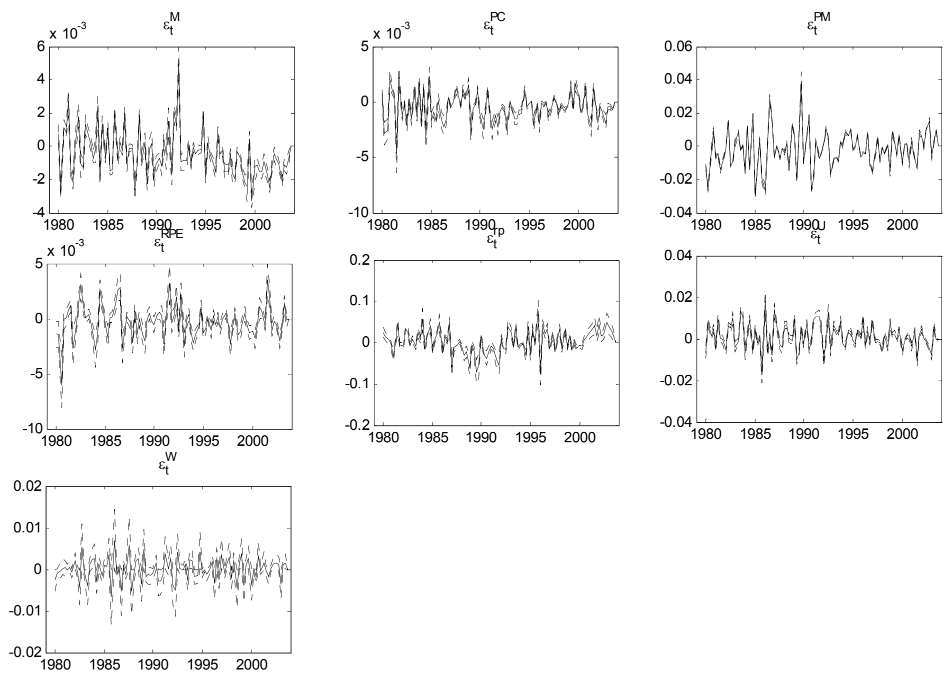

Figure 13. Posterior mean (solid) and Highest Posterior Densitiy interval (dashed) of smoothed shocks.

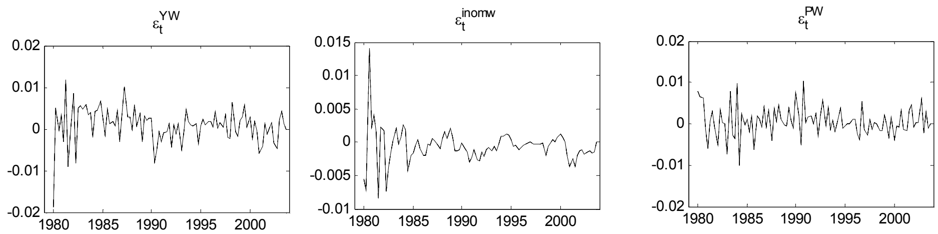

Figure 14. Posterior mean (solid) and Highest Posterior Densitiy interval (dashed) of smoothed shocks.

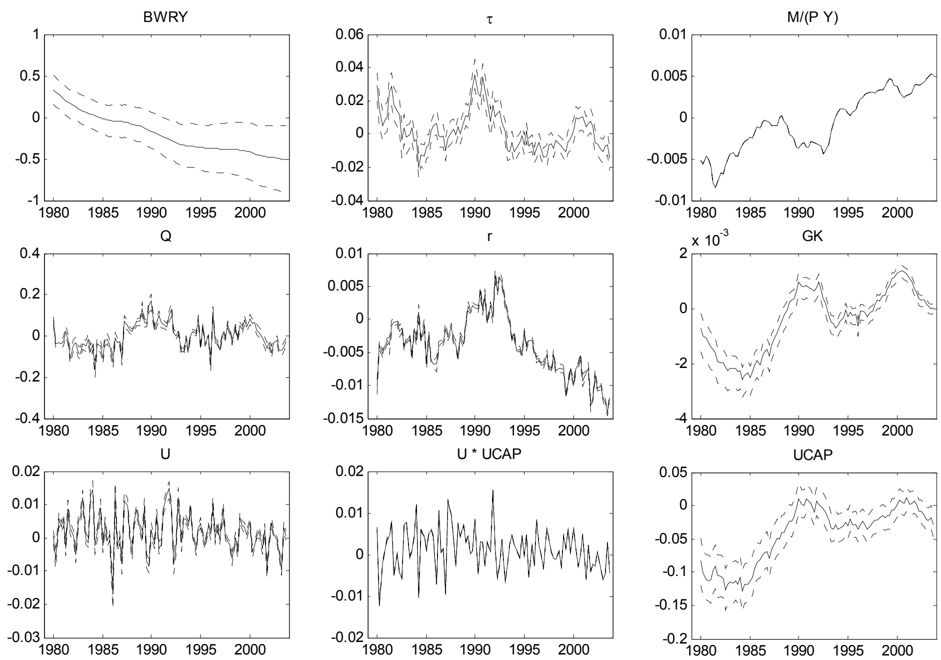

Figure 15. Posterior mean (solid) and Highest Posterior Densitiy interval (dashed) of smoothed unobserved variables.[2]

[2] Growth rate variables have the 'G' prefix: e.g. GC is growth rate of consumption.

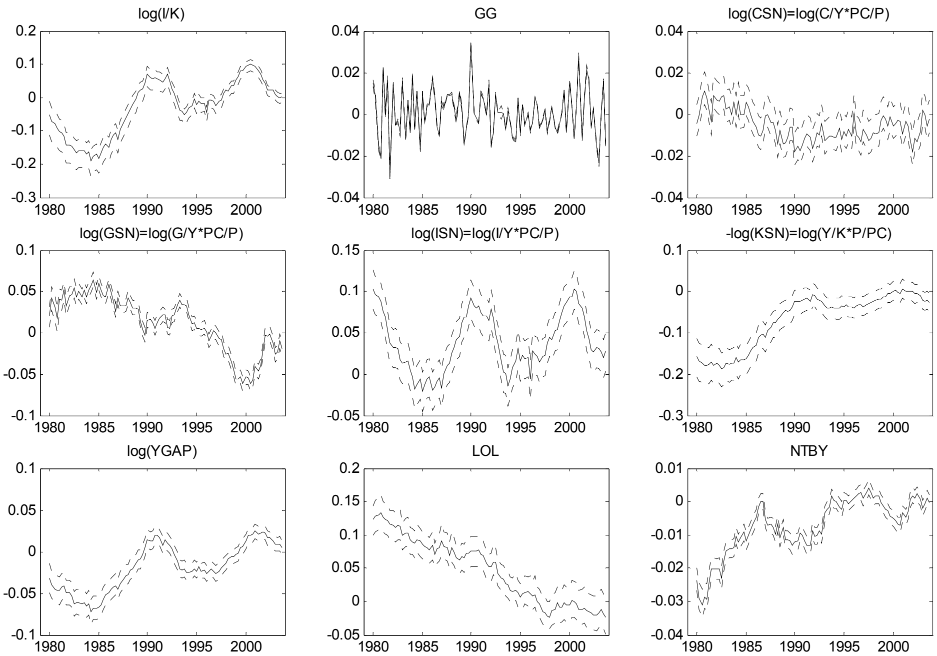

Figure 16. Posterior mean (solid) and Highest Posterior Densitiy interval (dashed) of smoothed unobserved variables.

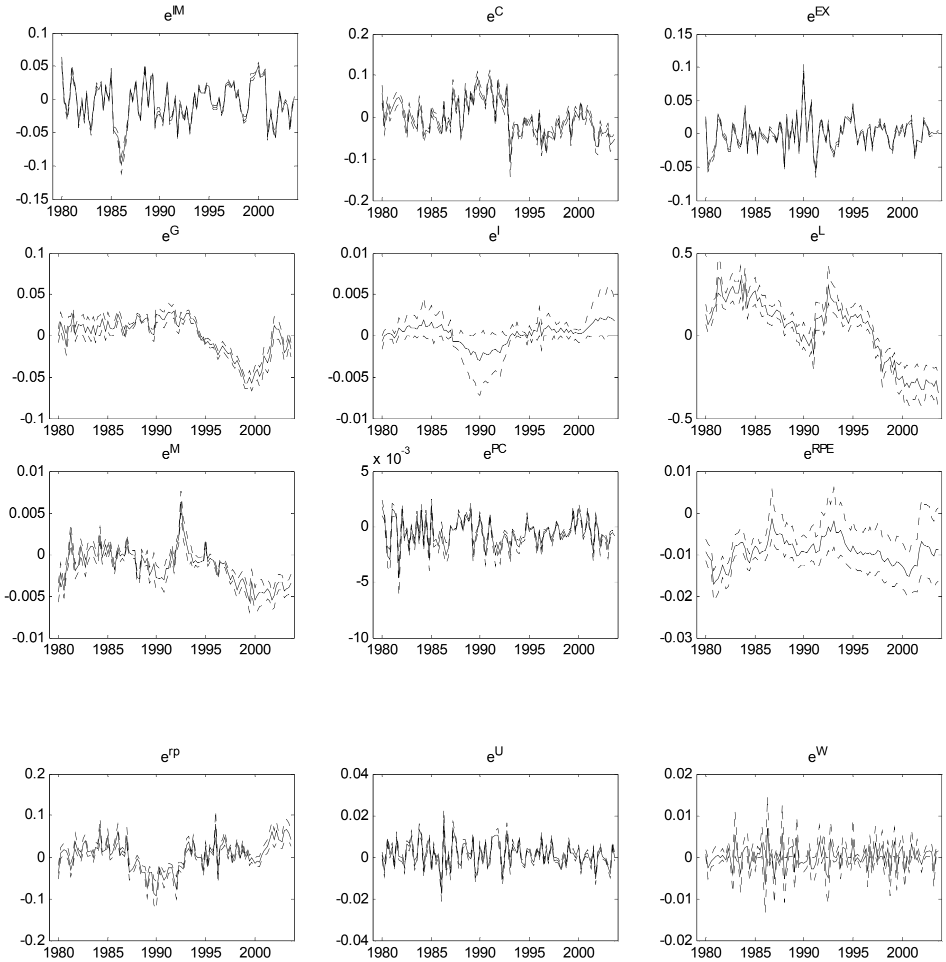

Figure 17. Posterior mean (solid) and Highest Posterior Densitiy interval (dashed) of smoothed unobserved variables.

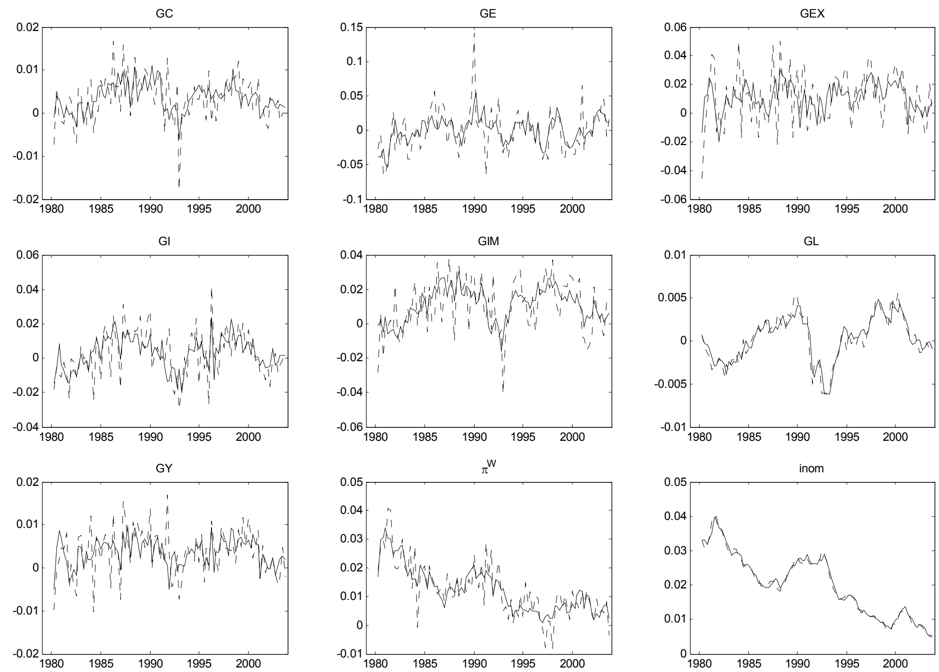

Figure 18. VECM fit: solid lines are 1-step ahead predictions, while dashed lines are observations.

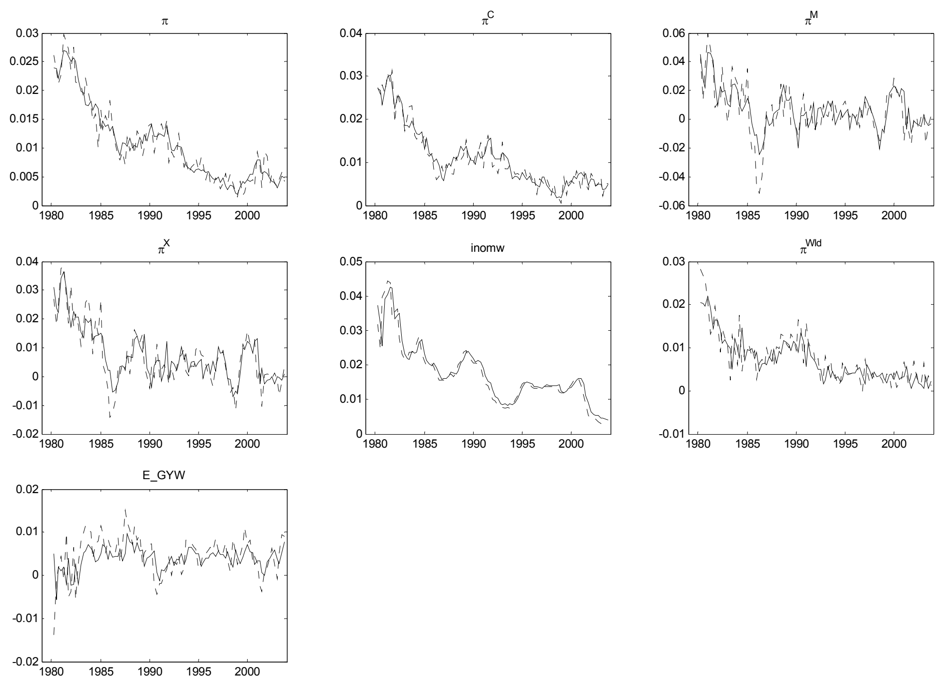

Figure 19. VECM fit: solid lines are 1-step ahead predictions, while dashed lines are observations.

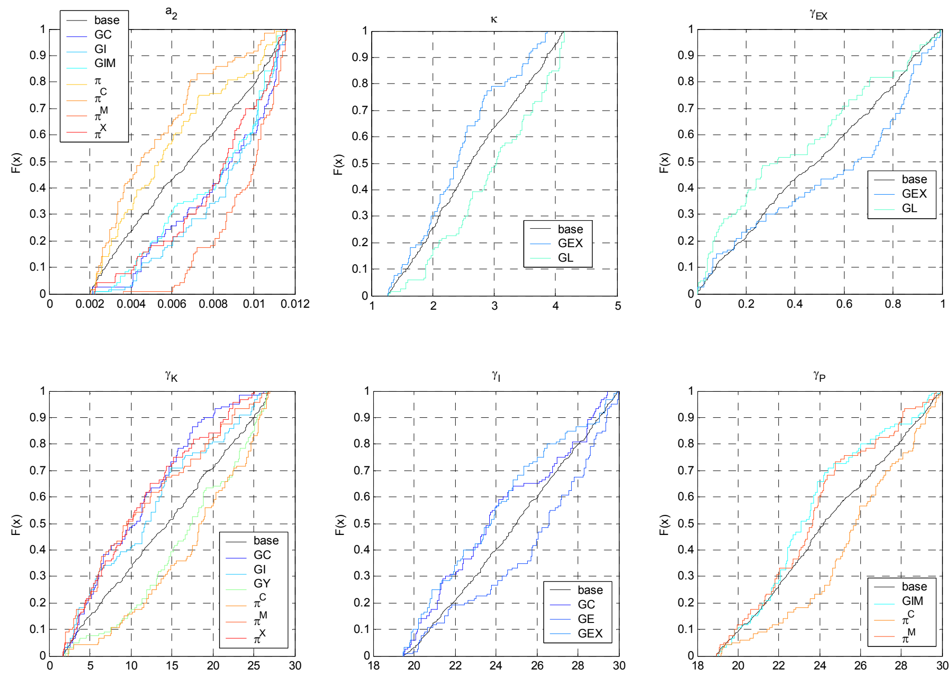

Figure 20

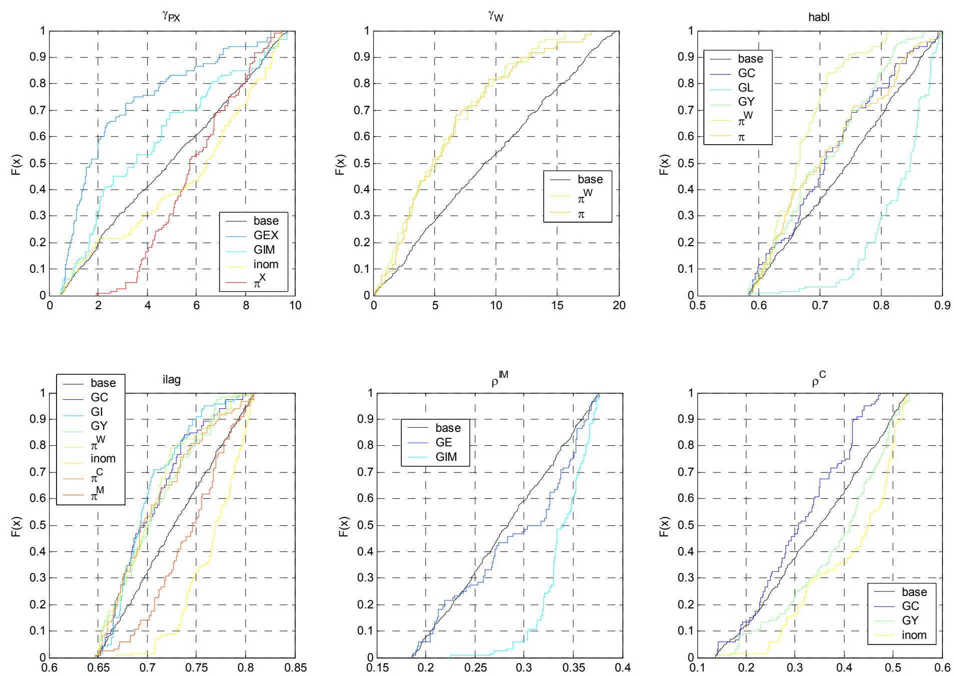

Figure 21

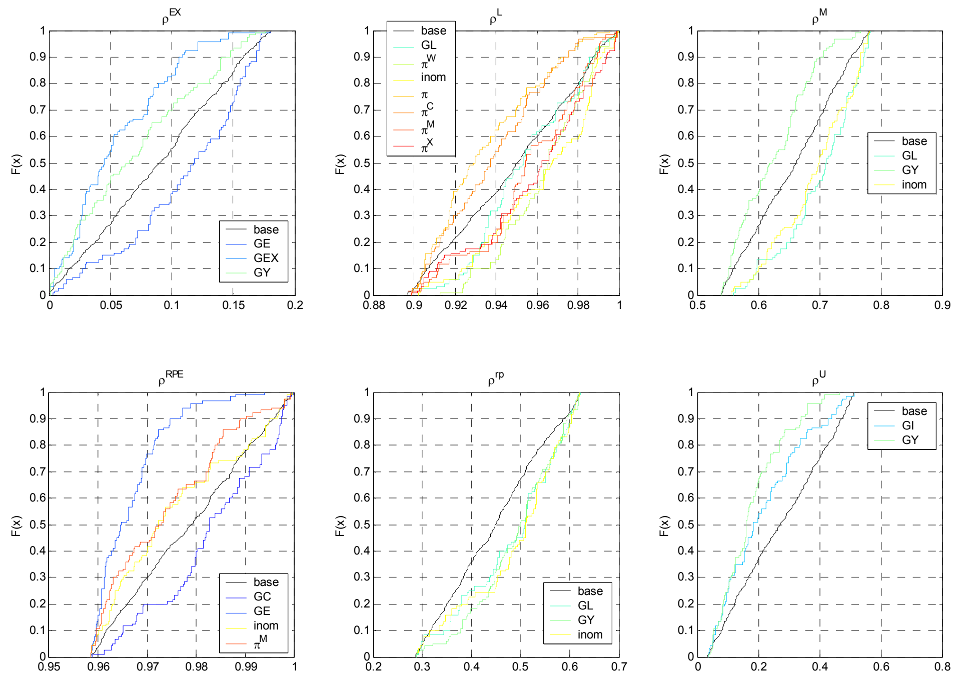

Figure 22

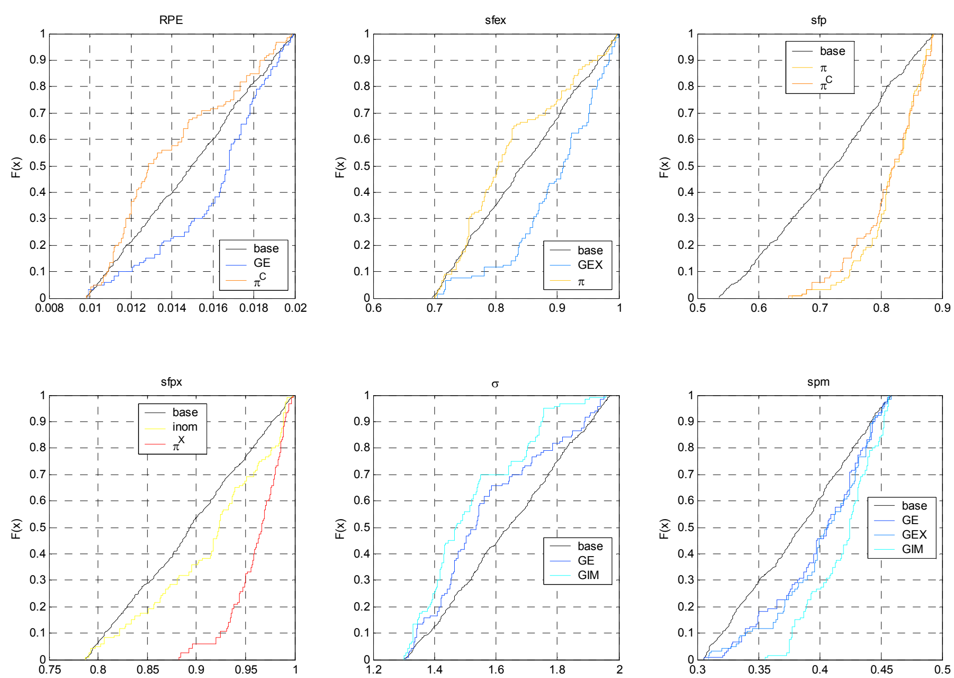

Figure 23

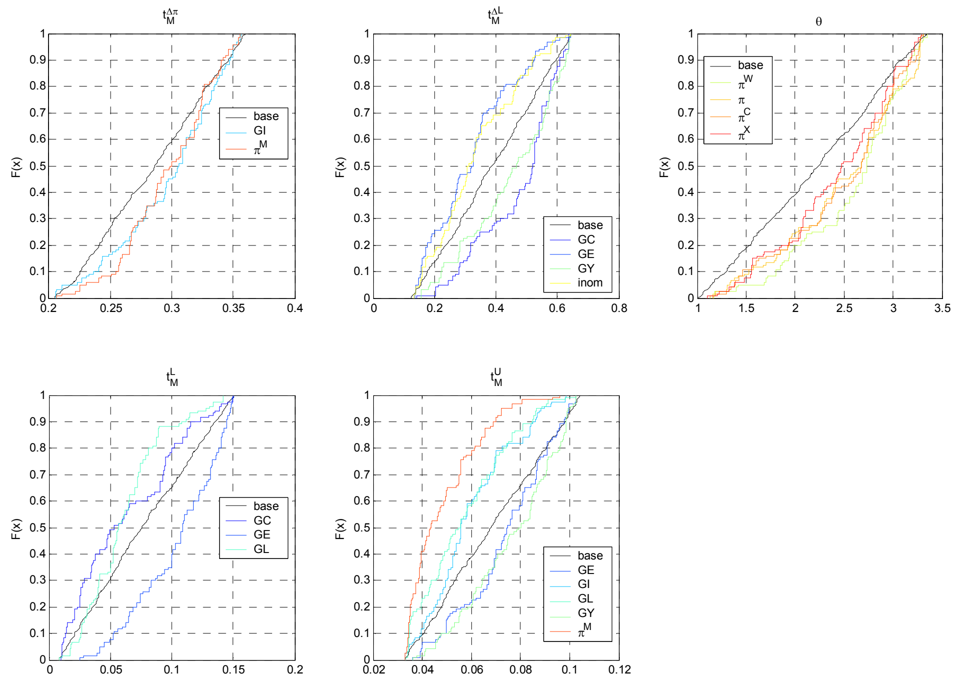
$t_M^{\Delta\pi}$
base
GI
π^M
$t_M^{\Delta L}$
base
GC
GE
GY
inom
θ
base
π^W
π
π^C
π^X
t_M^L
base
GC
GE
GL
t_M^U
base
GE
GI
GL
GY
π^M
F(x)

Figure 24

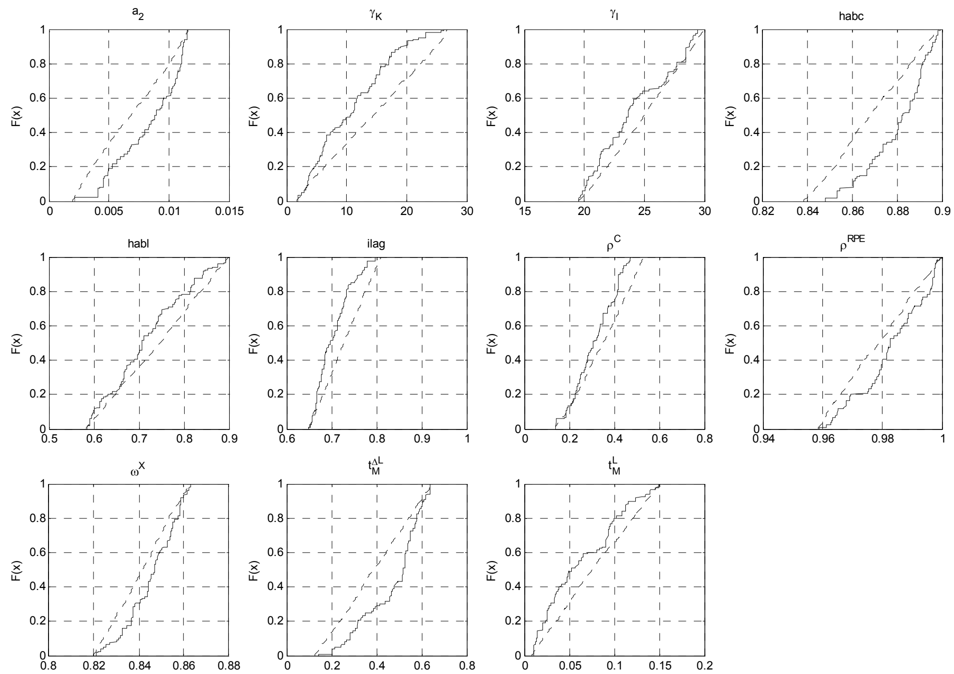

Figure 25. Parameters mainly affecting the fit of GC.

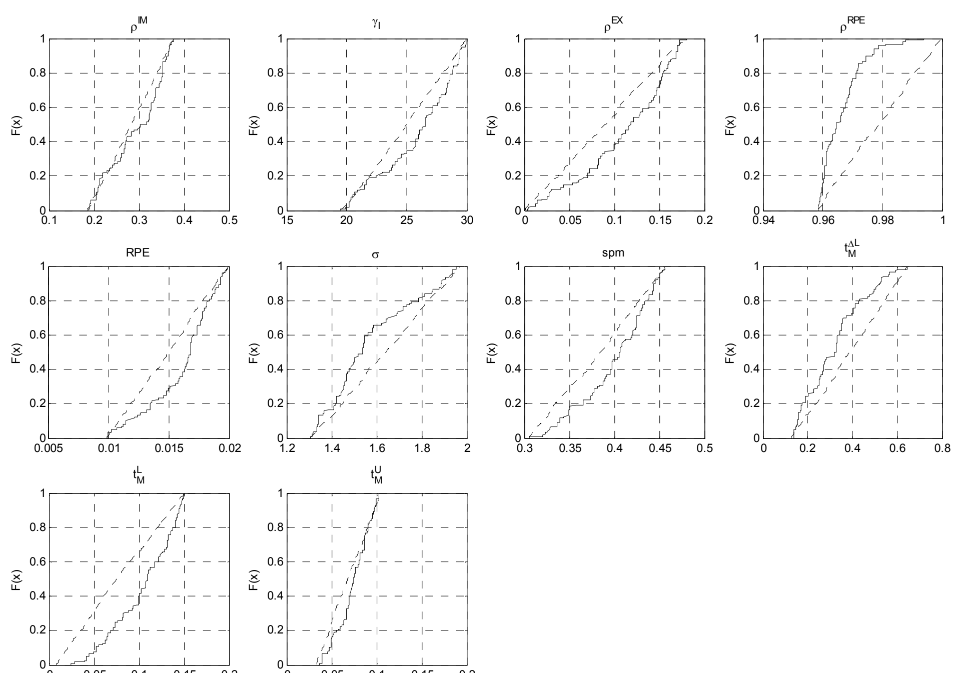

Figure 26. Parameters mainly affecting the fit of GE.

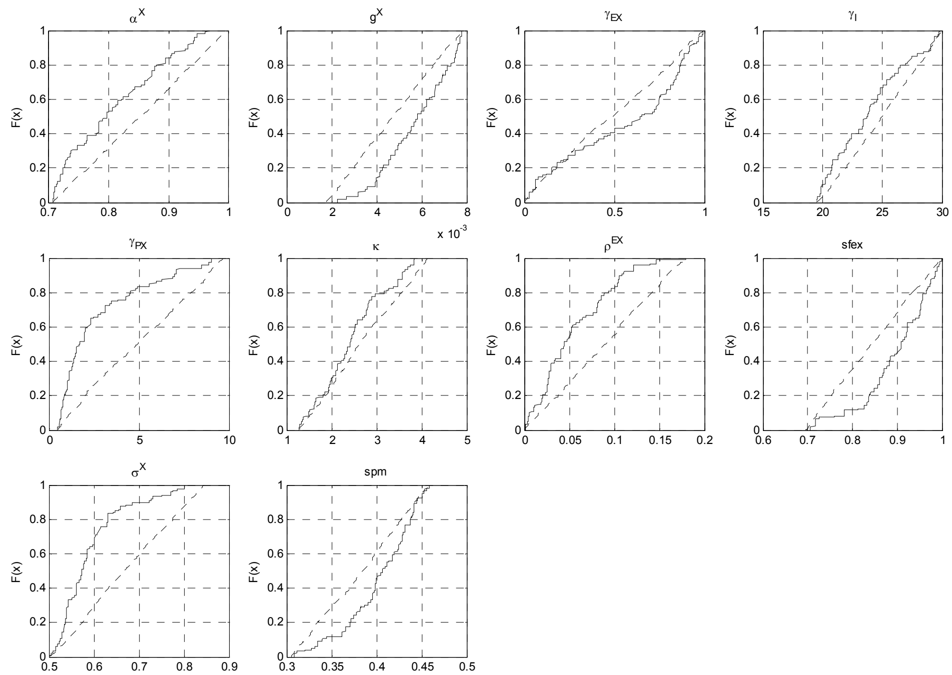

Figure 27. Parameters mainly affecting the fit of GEX.

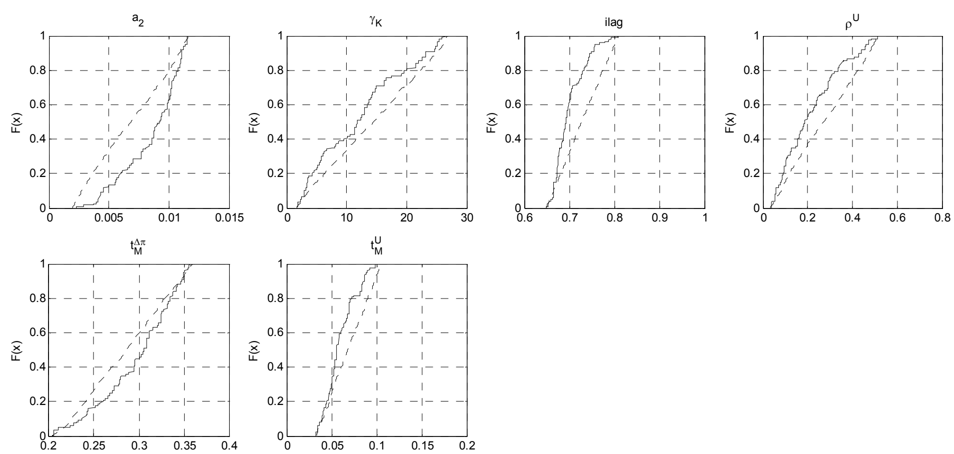

Figure 28. Parameters mainly affecting the fit of GI.

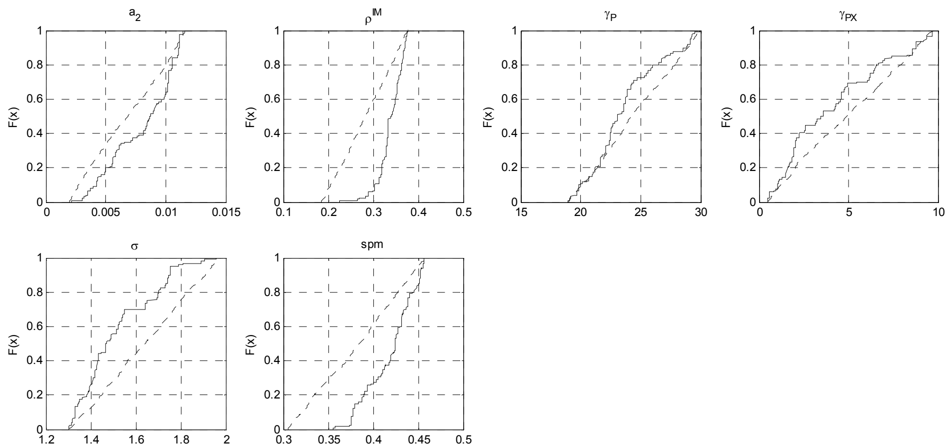

Figure 29. Parameters affecting the fit of GIM.

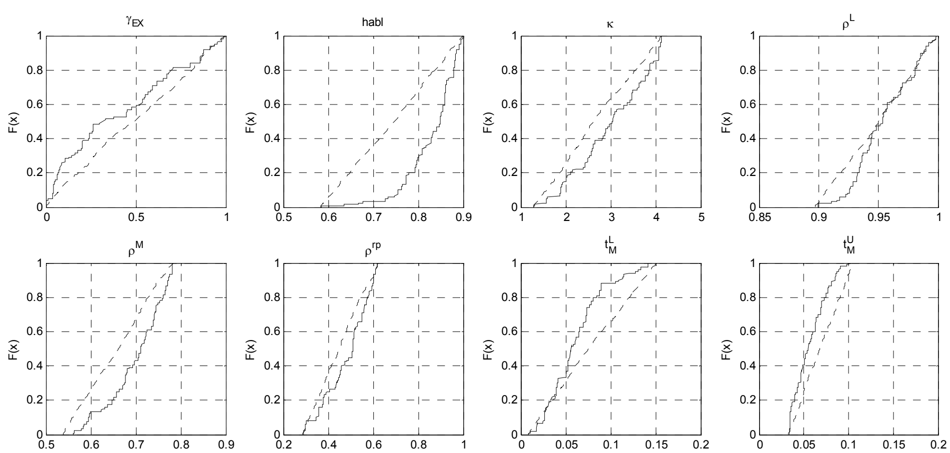

Figure 30. Parameters affecting the fit of GL.

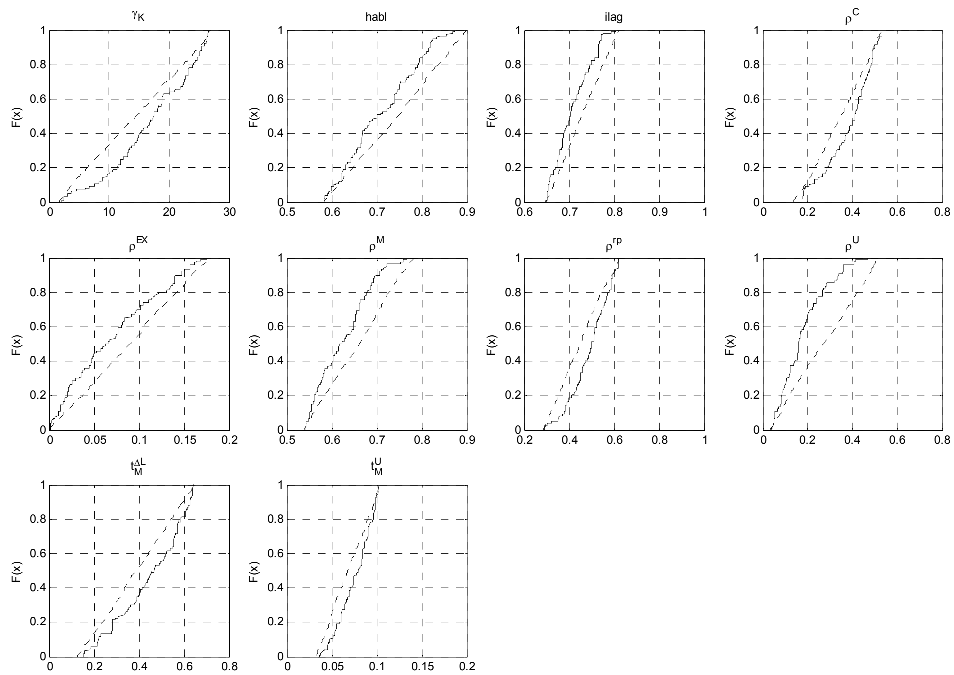

Figure 31. Parameters affecting the fit of GY.

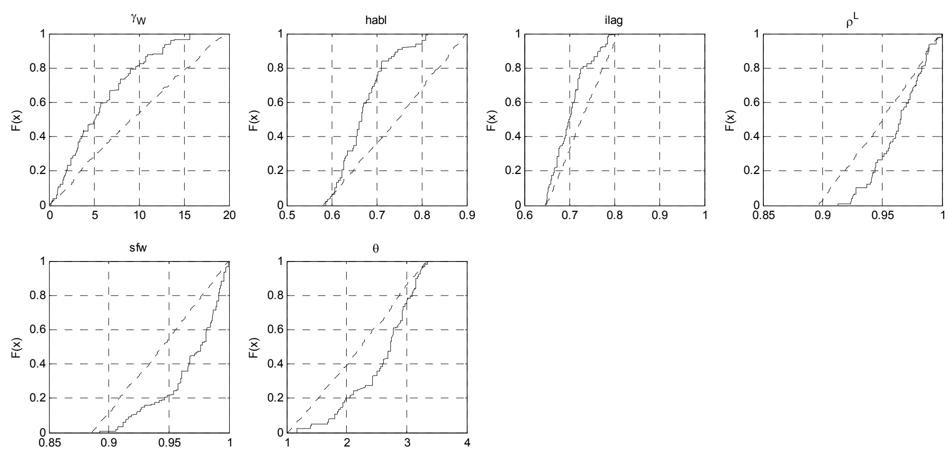

Figure 32. Parameters affecting the fit of π^W.

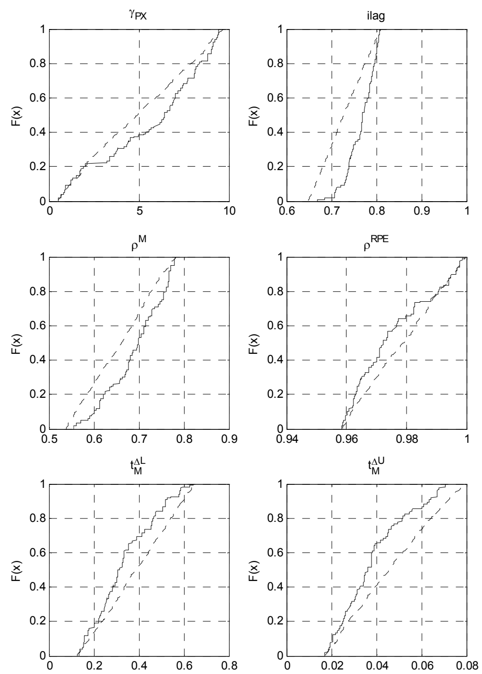

Figure 33. Parameters affecting the fit of inom.

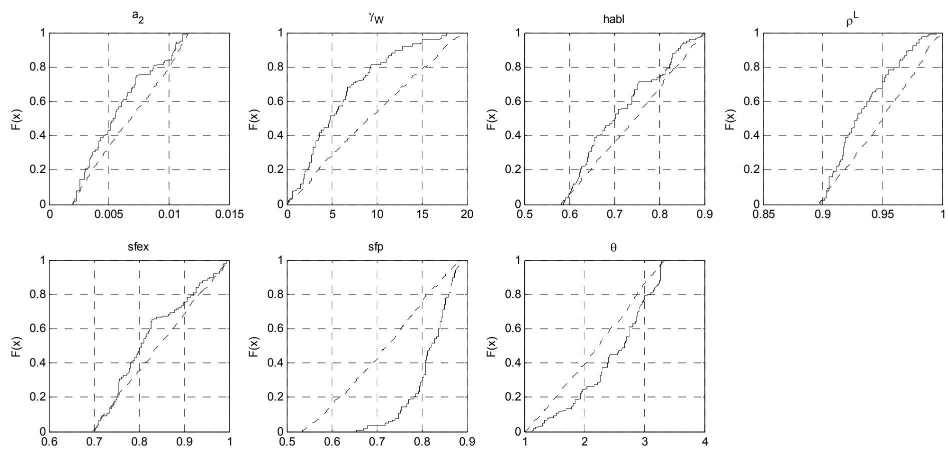

Figure 34. Parameters affecting the fit of π .

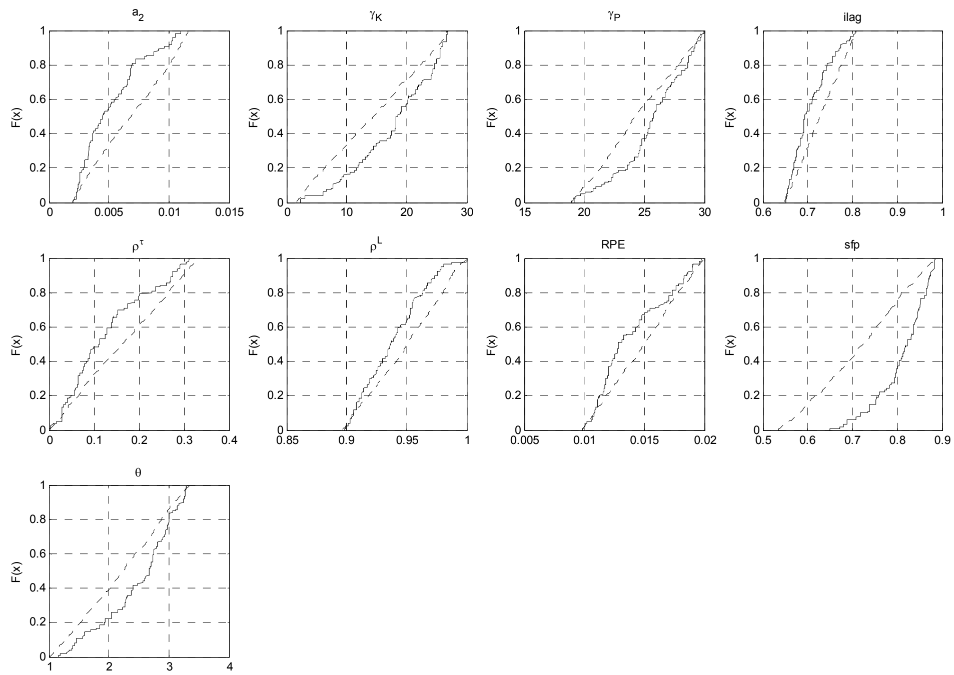

Figure 35. Parameters affecting the fit of π^C.

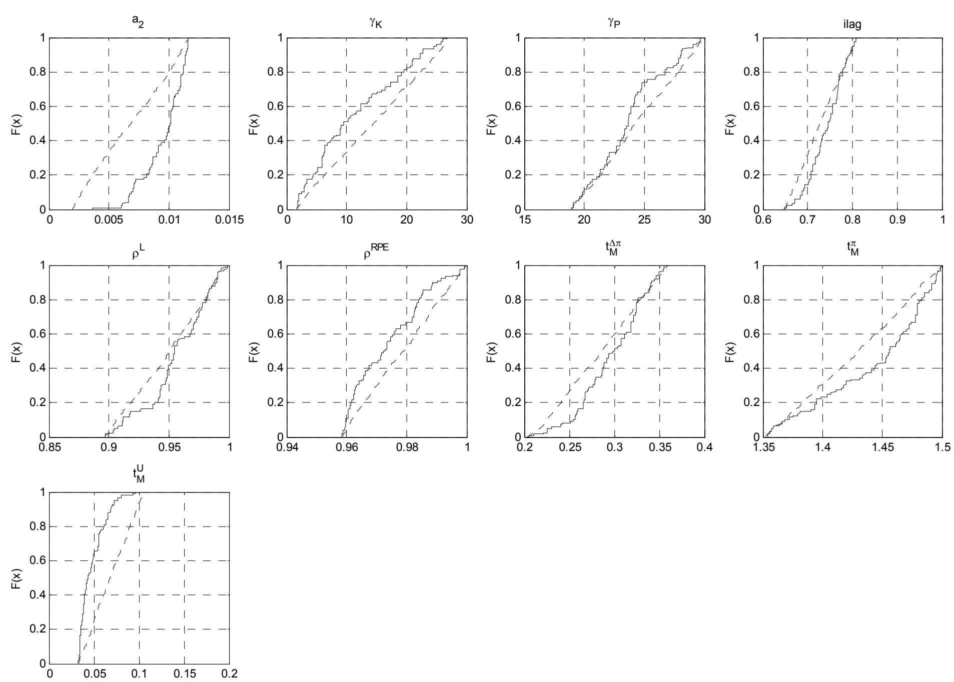

Figure 36. Parameters affecting the fit of π^M.

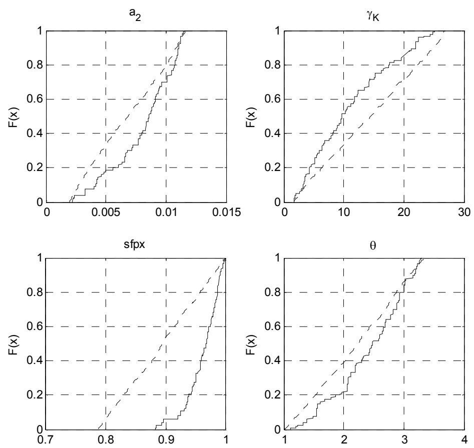

Figure 37. Parameters affecting the fit of π^X.

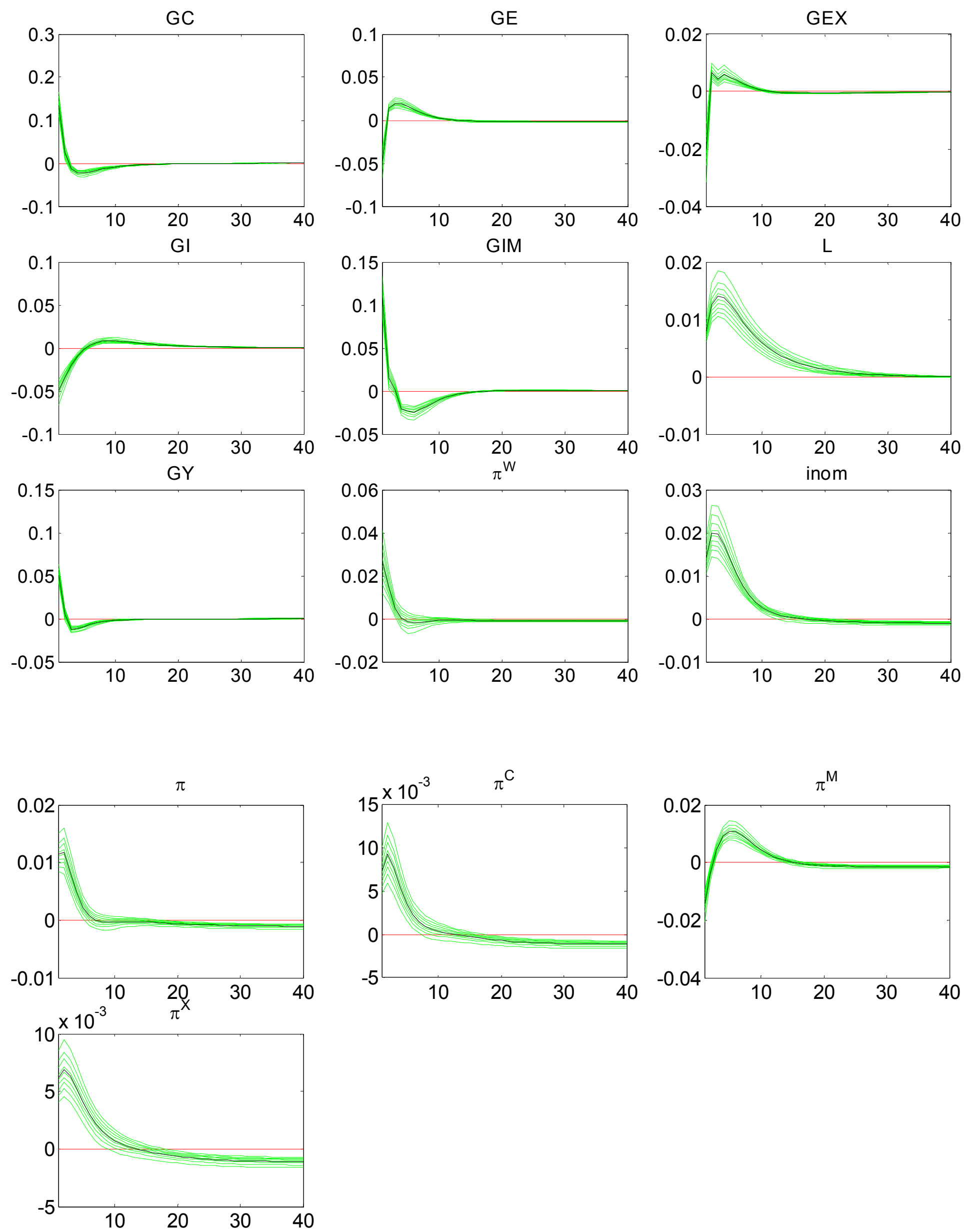

Figure 38. Unit shock to ε^C.

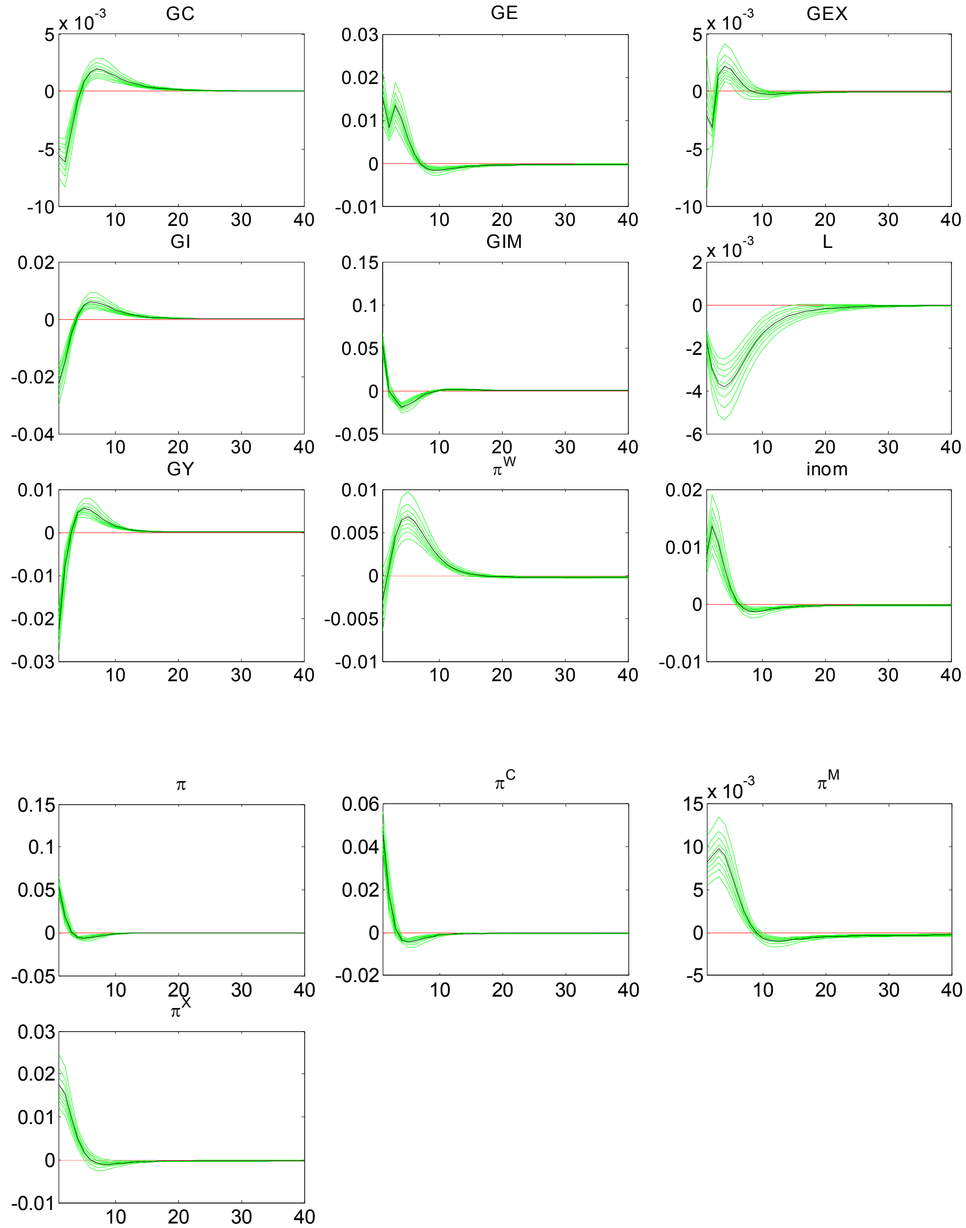

Figure 39. Unit shock to ε^{τ}.

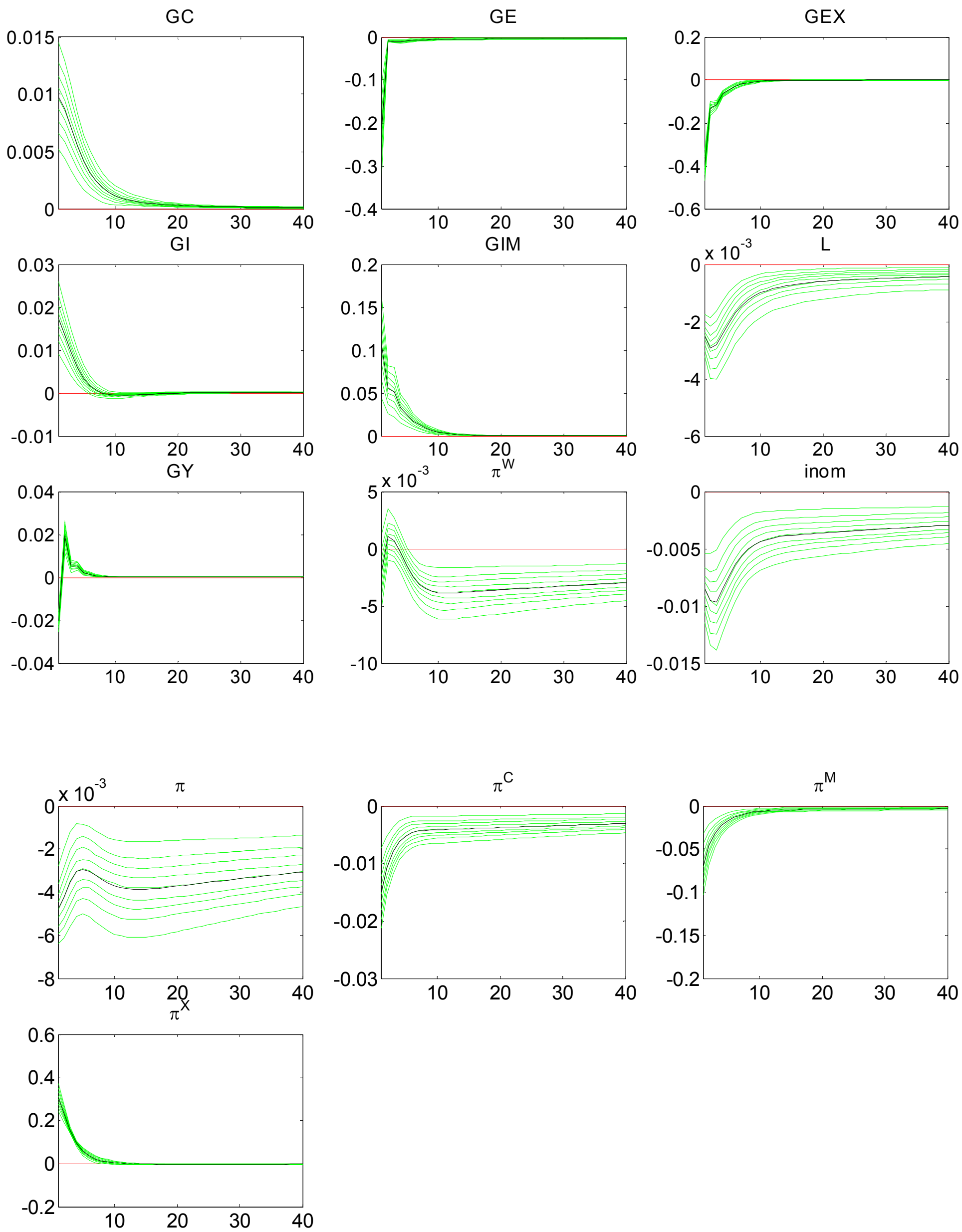

Figure 40. Unit shock to $\varepsilon^{\tau X}$.

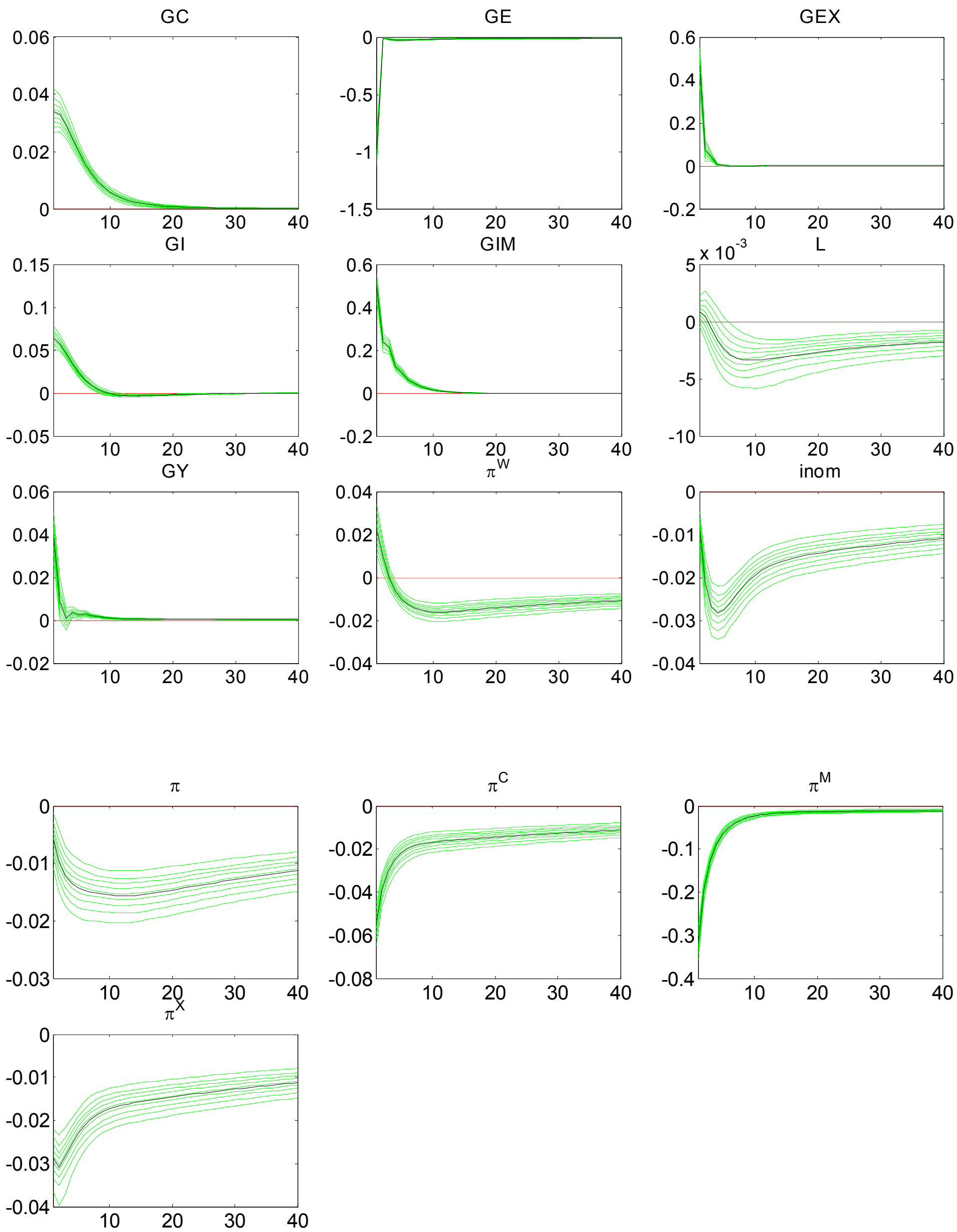

Figure 41. Unit shock to ε^{EX} **.**

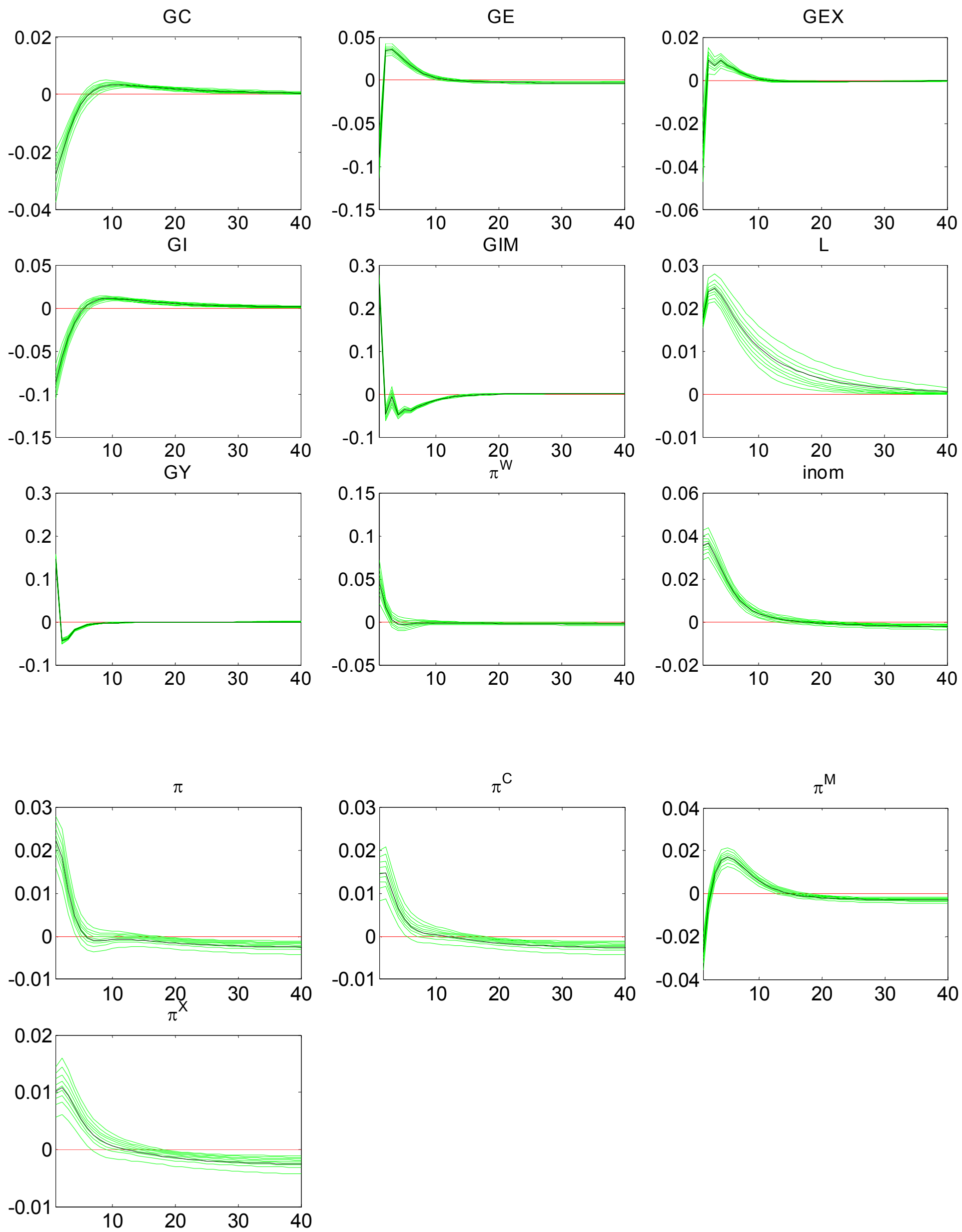

Figure 42. Unit shock to ε^G.

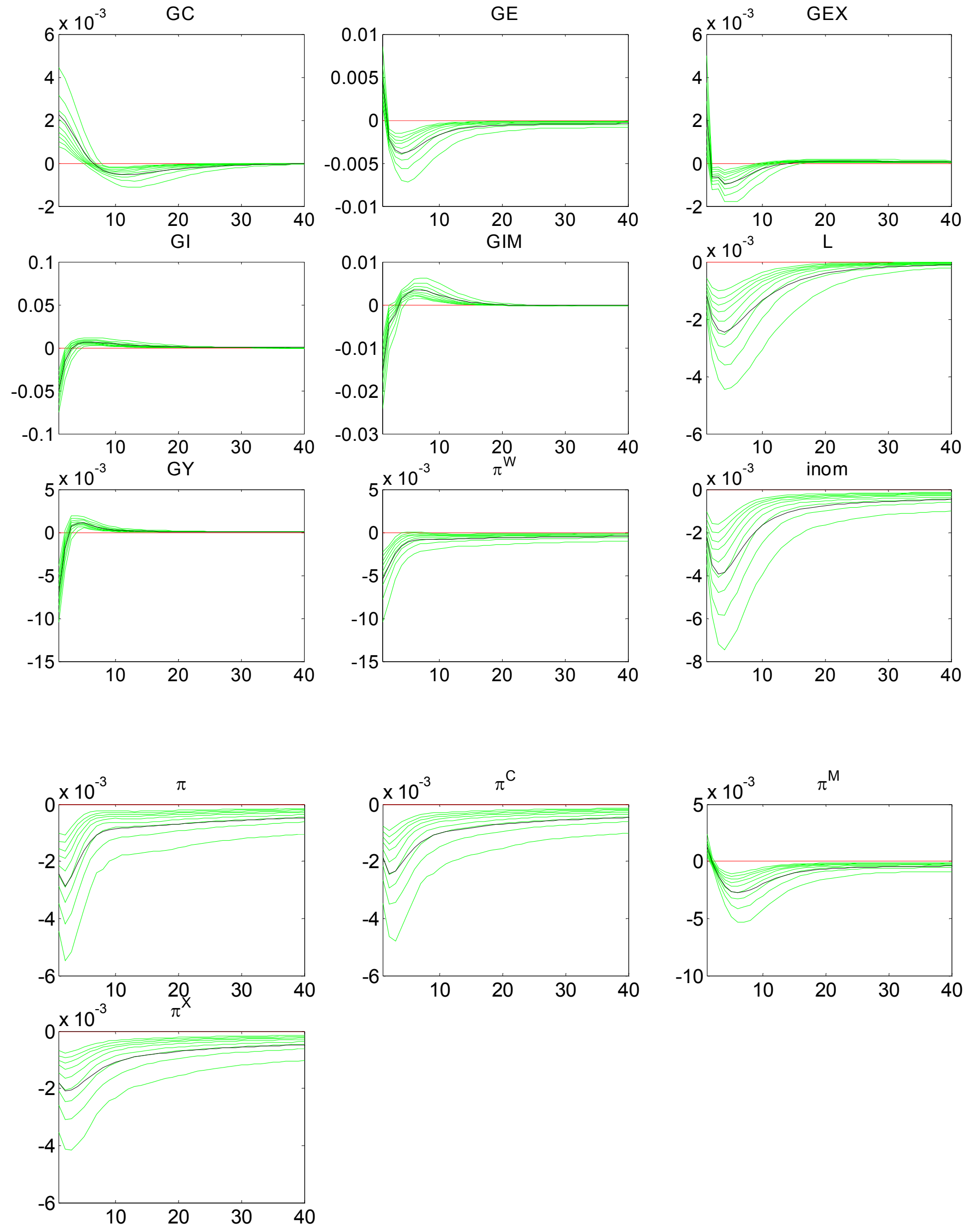

Figure 43. Unit shock to ε^I.

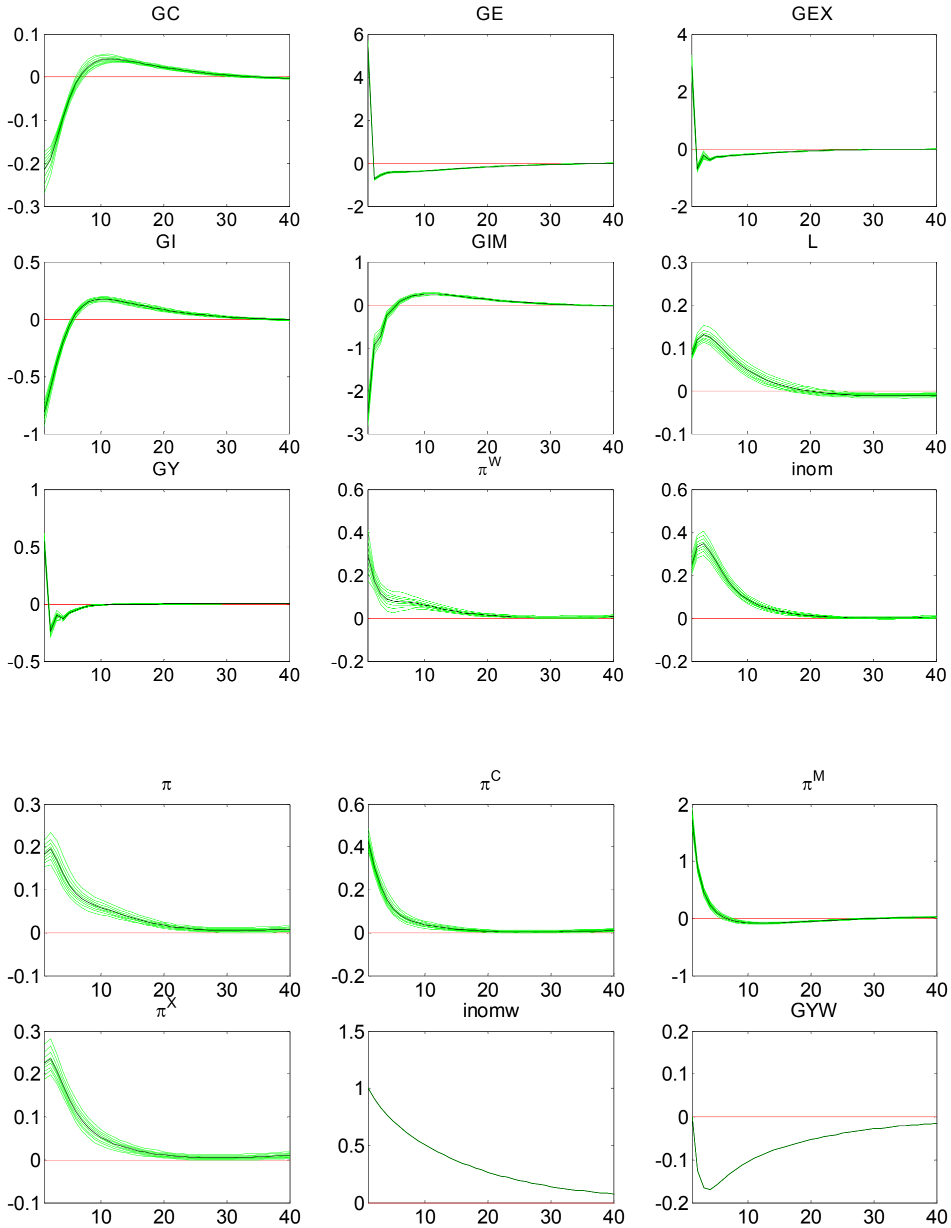

Figure 44. Unit shock to ε^{inomw}.

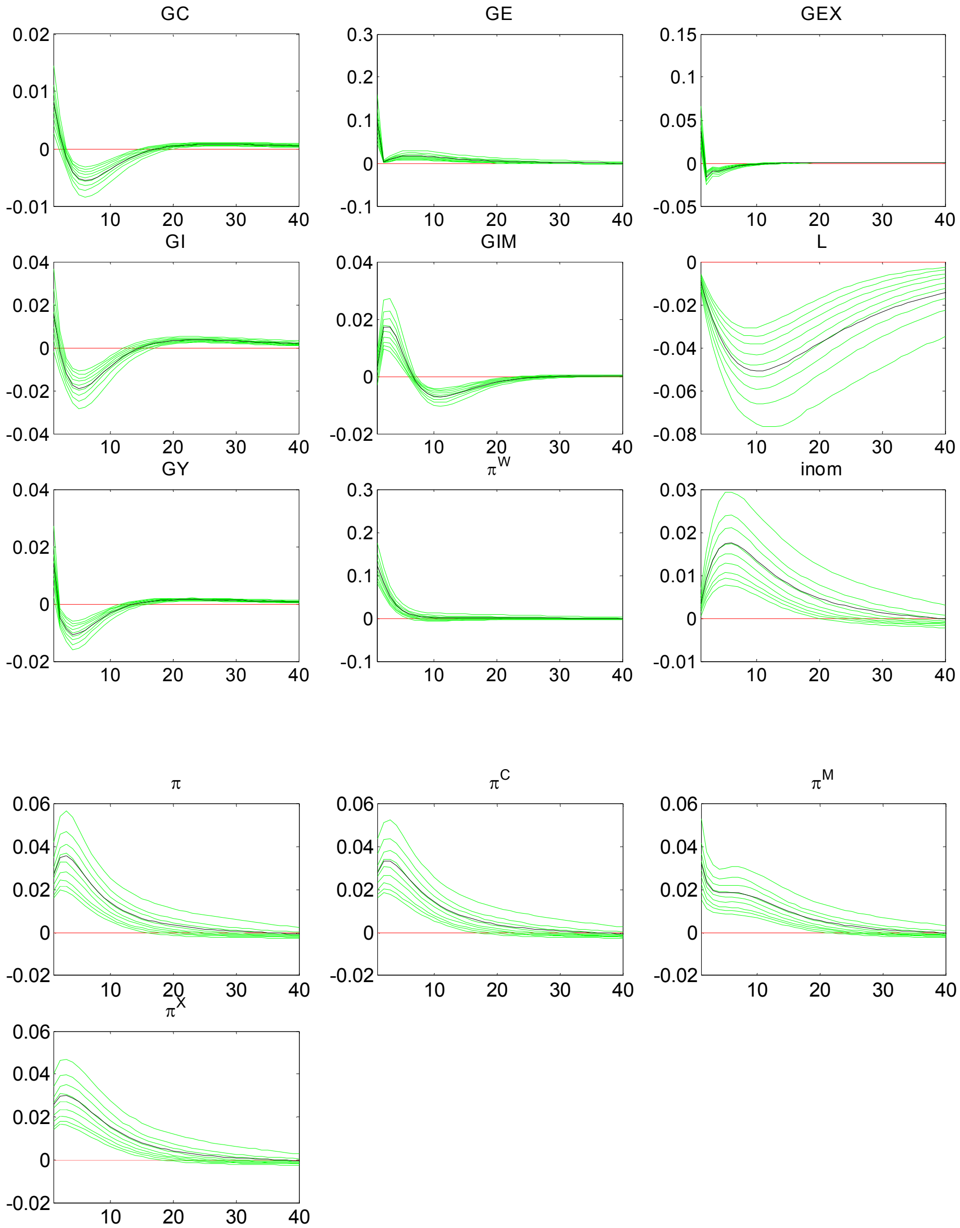

Figure 45. Unit shock to ε^L.

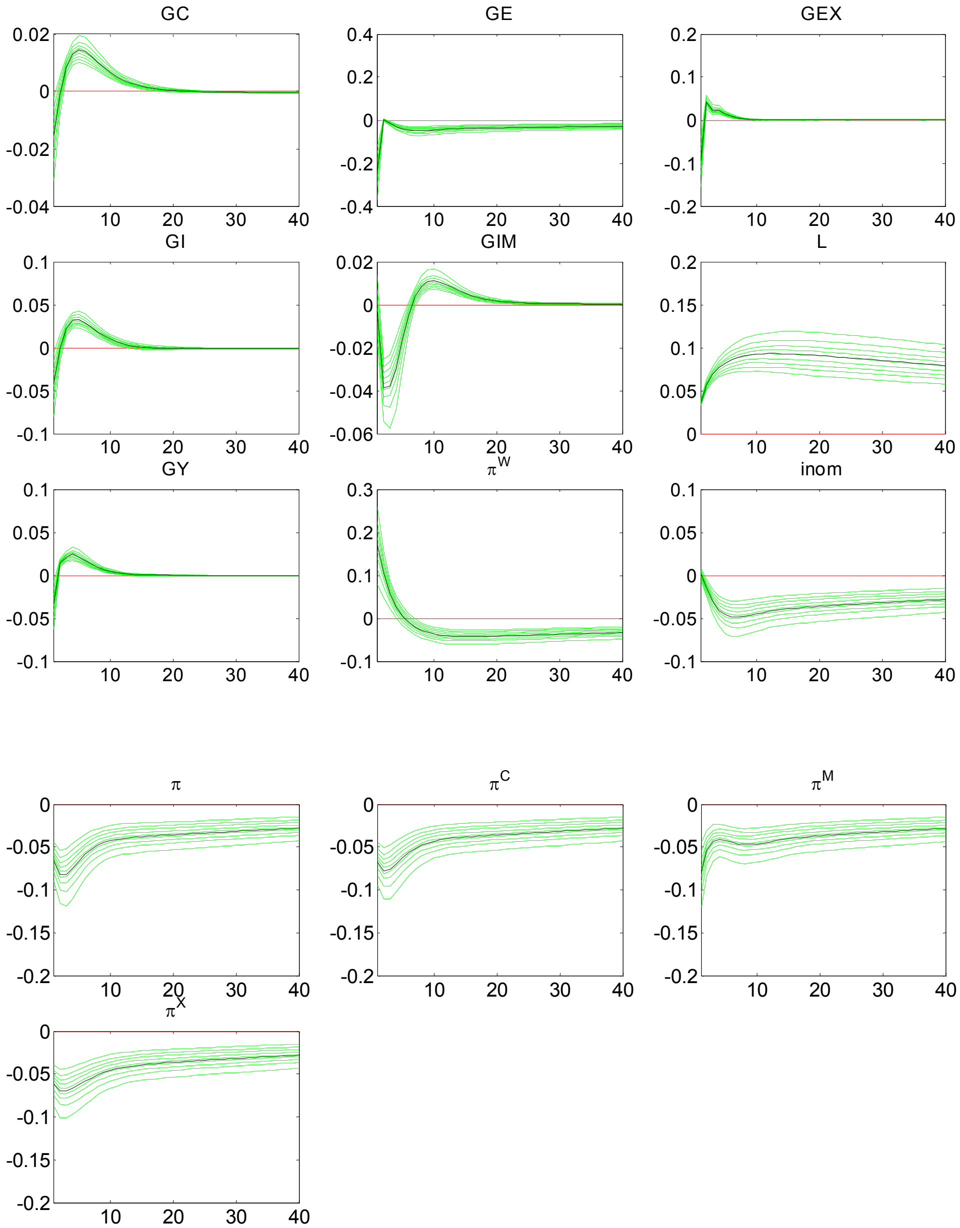

Figure 46. Unit shock to ε^{LOL}.

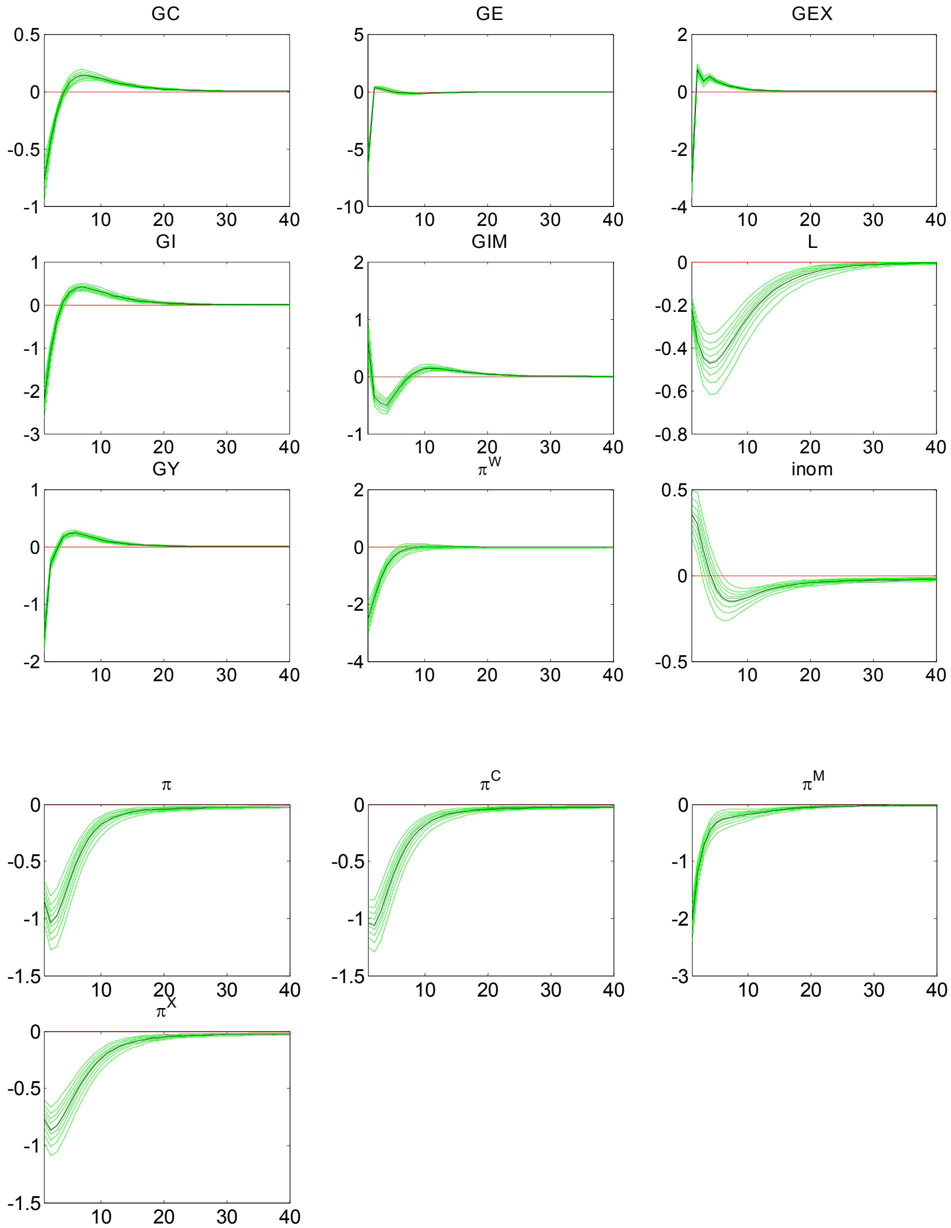

Figure 47. Unit shock to ε^M .

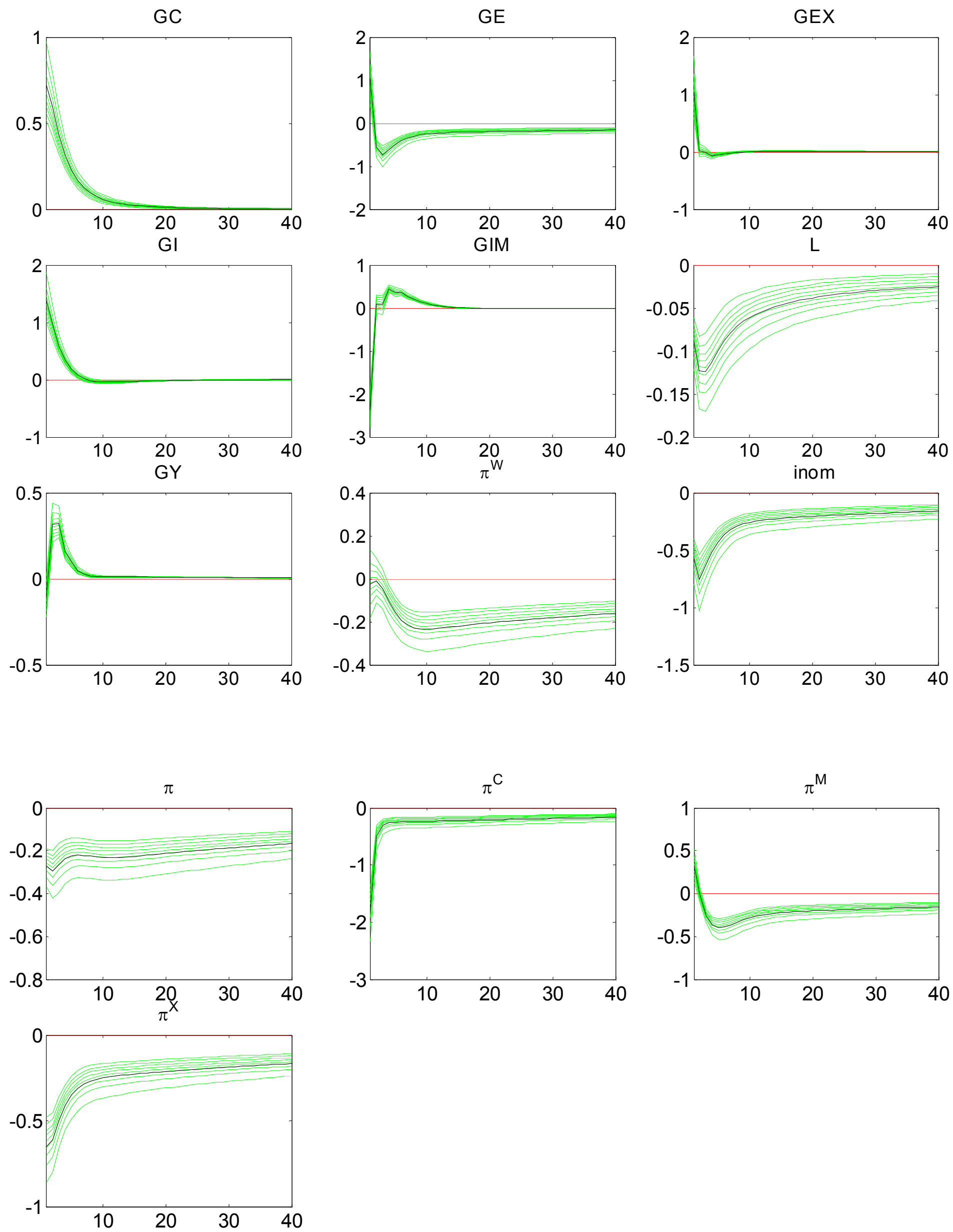

Figure 48. Unit shock to ε^{PC}.

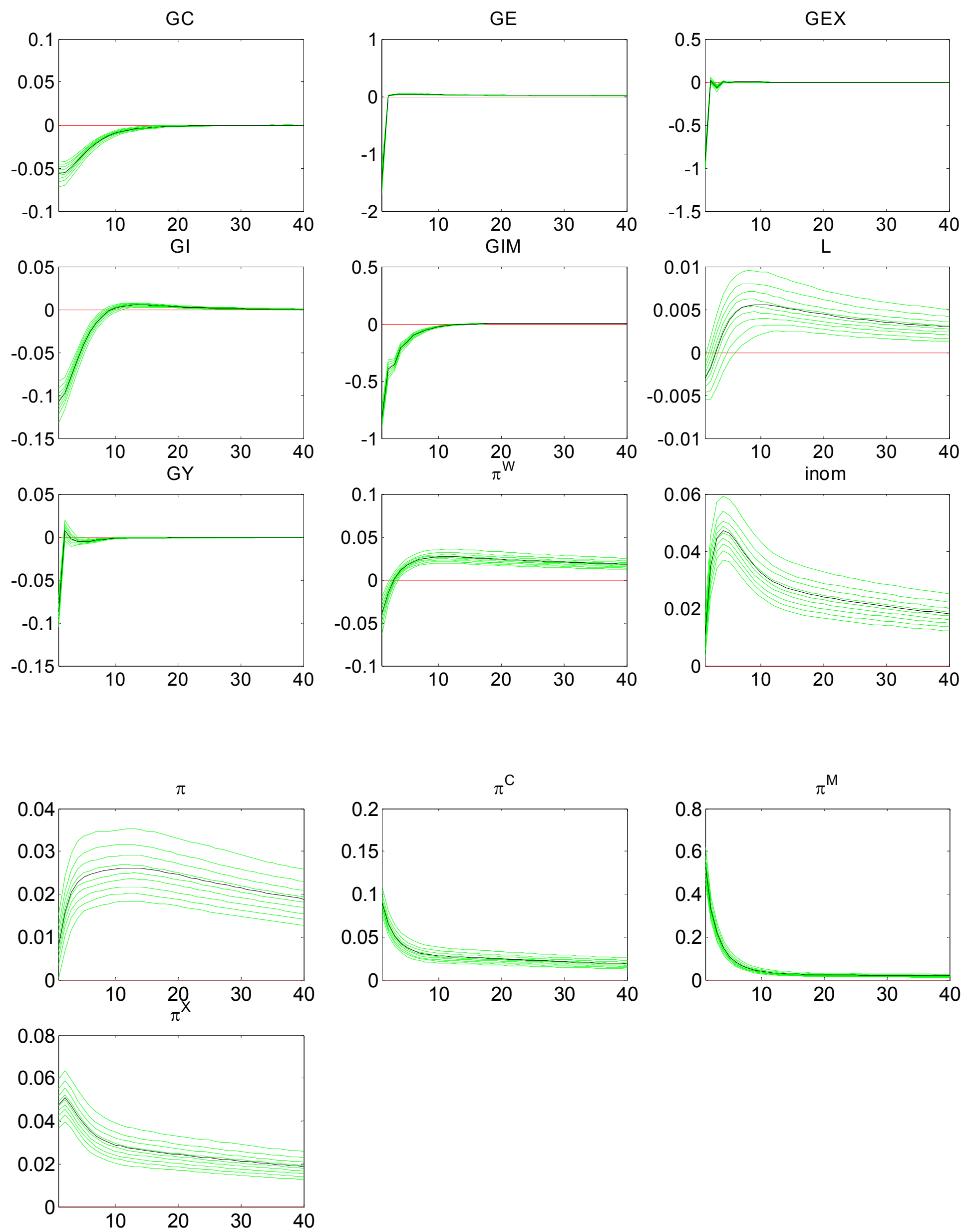

Figure 49. Unit shock to ε^{PM}.

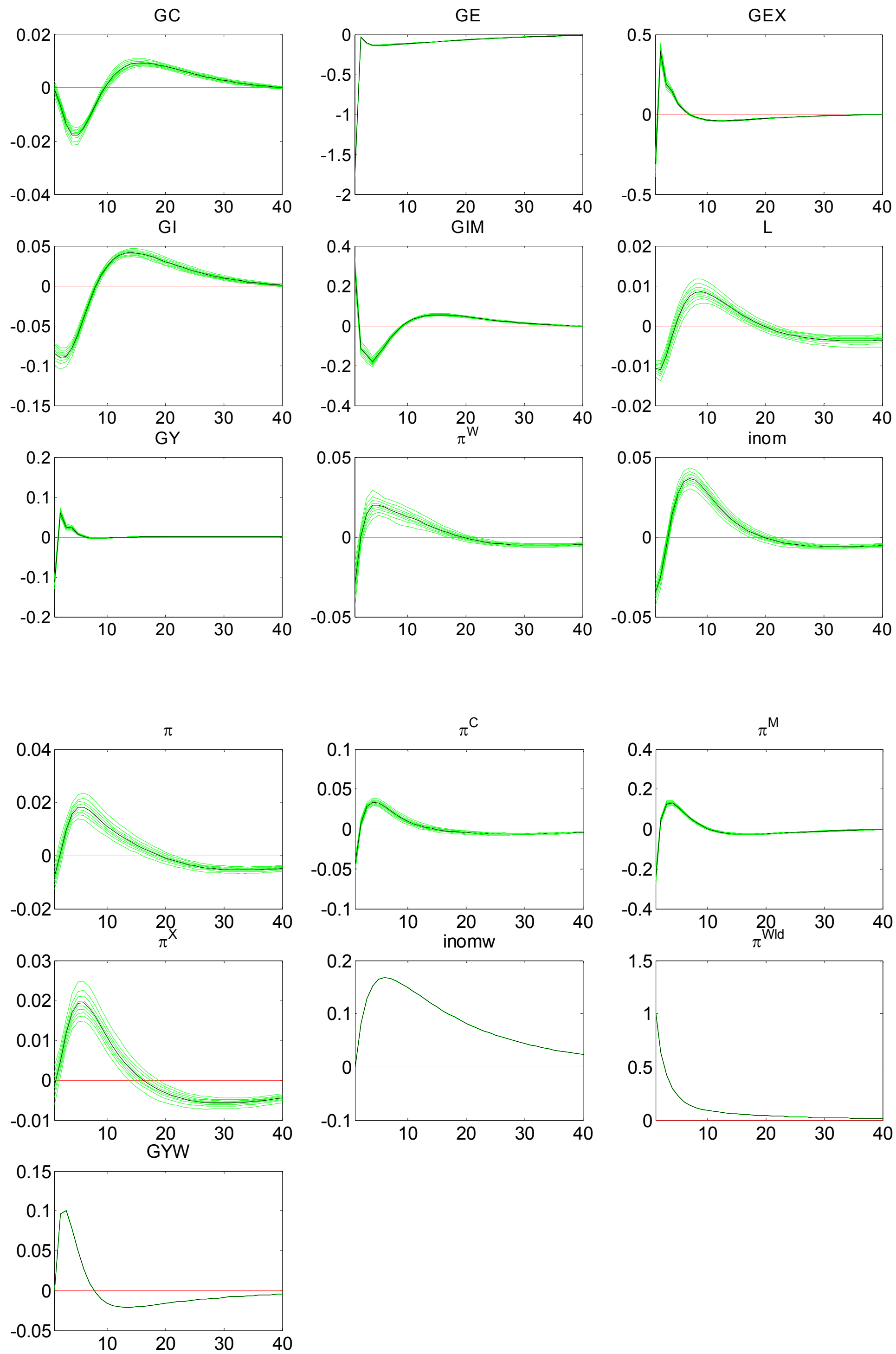

Figure 50. Unit shock to ε^{PW}.

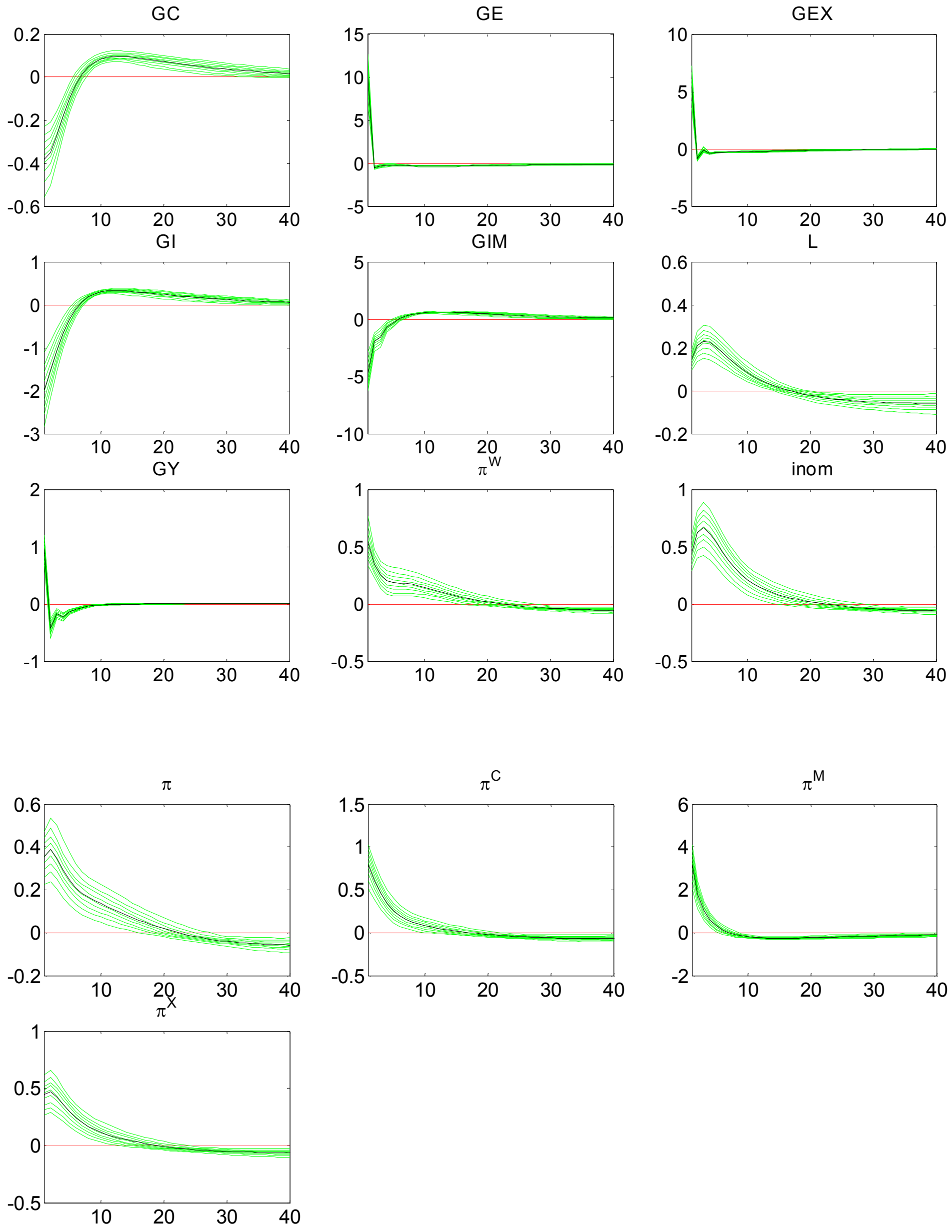

Figure 51. Unit shock ε^{RPE}.

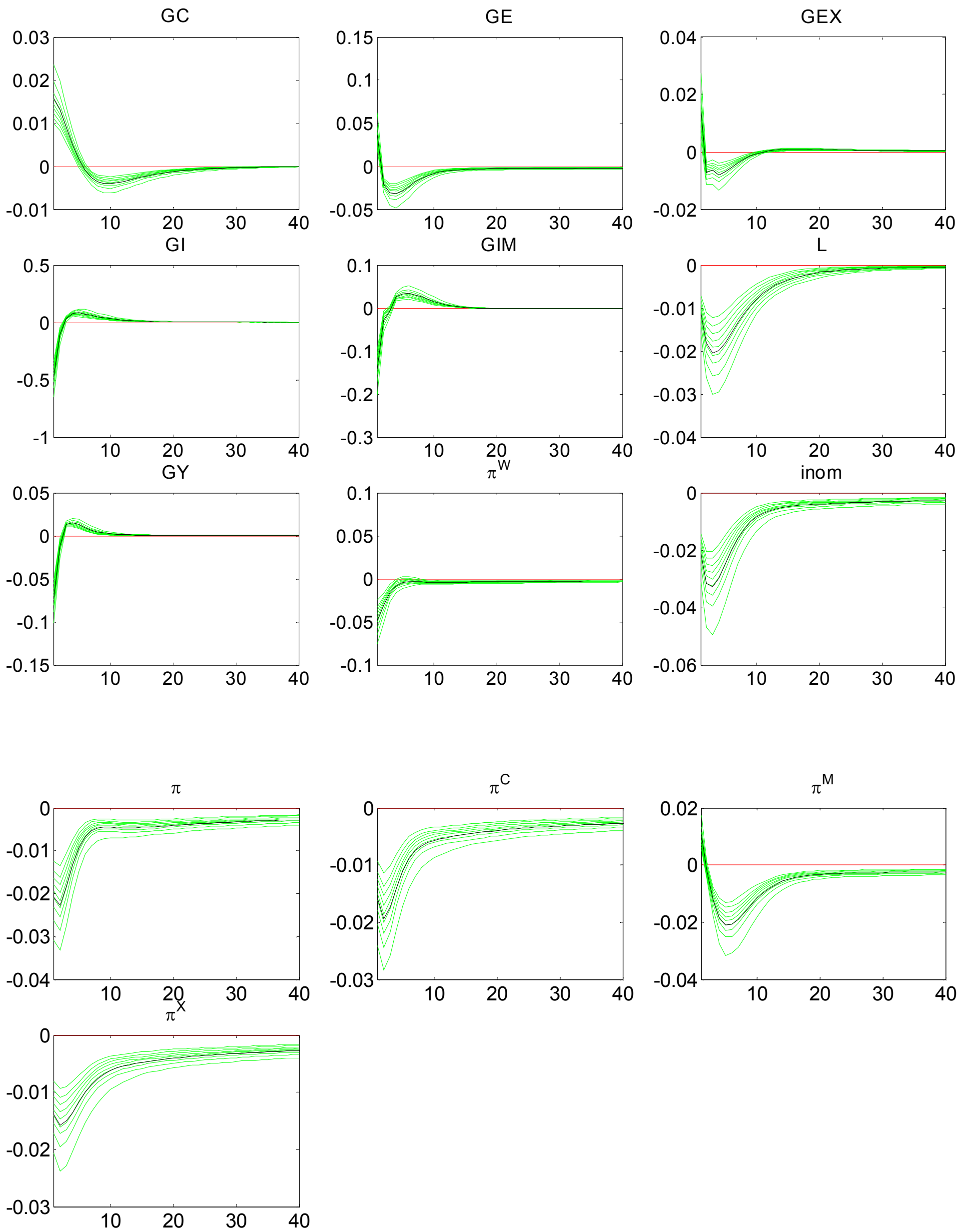

Figure 52. Unit shock to ε^{rp}.

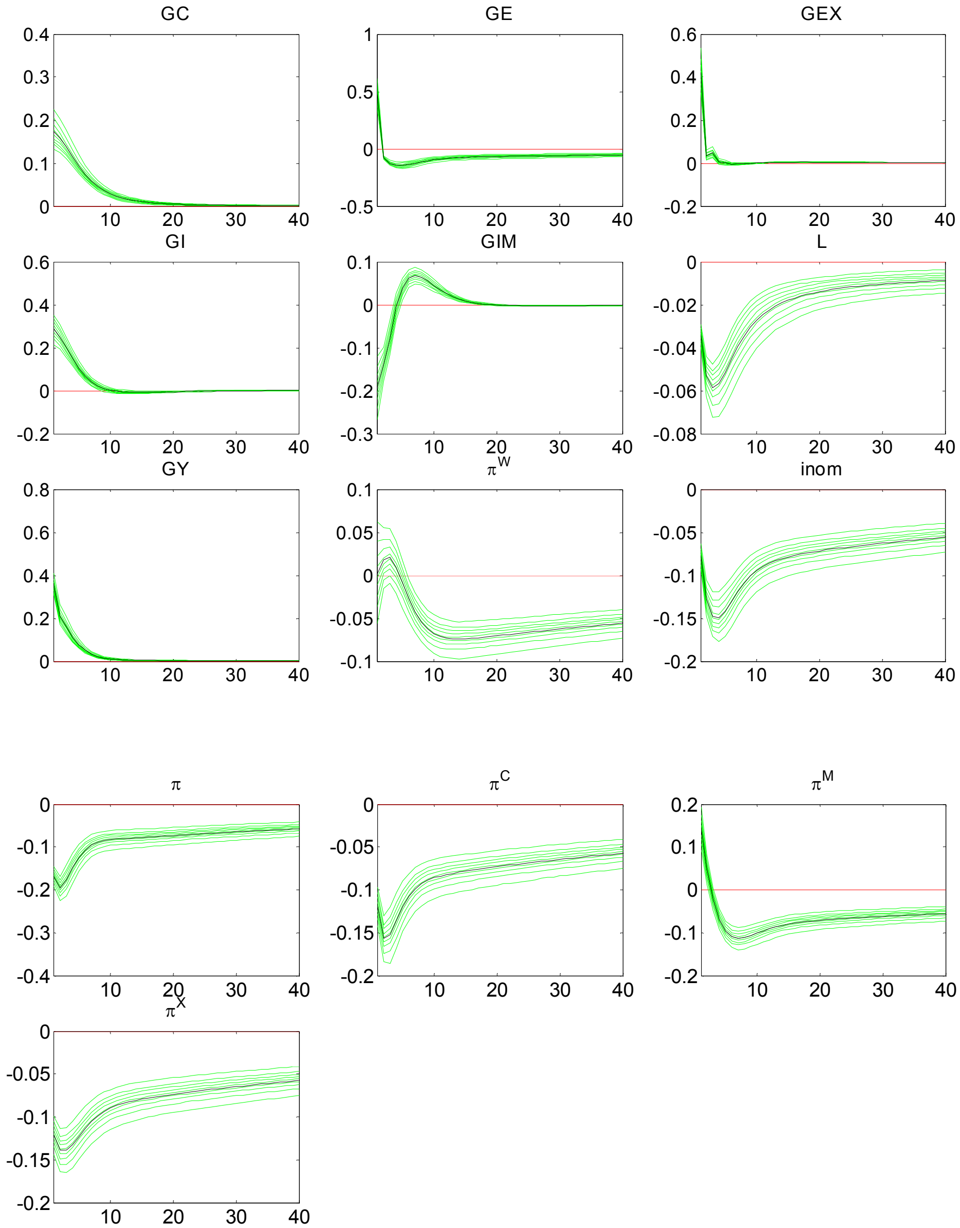

Figure 53. Unit shock to ε^{U} .

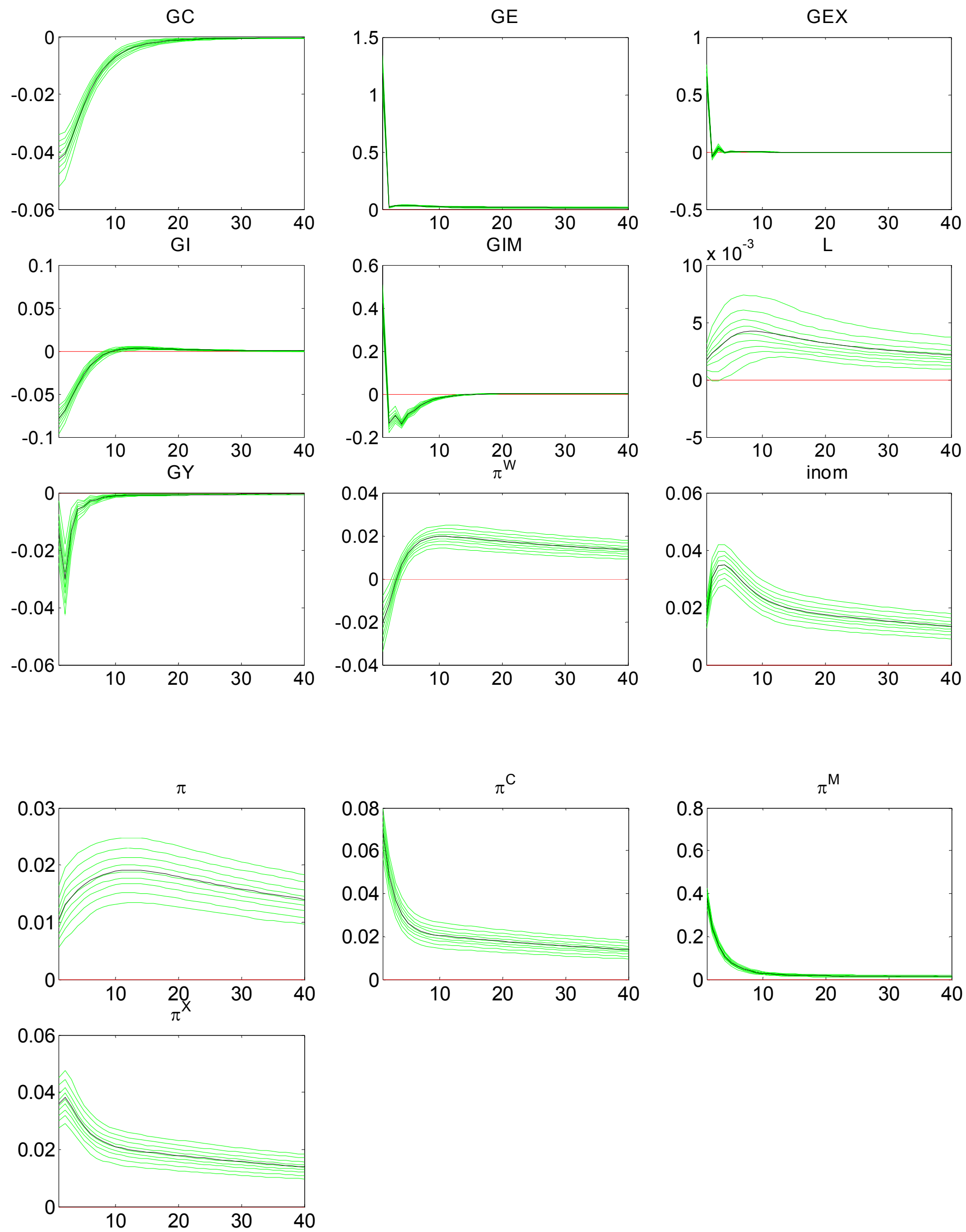

Figure 54. Unit shock to ε^{IM}.

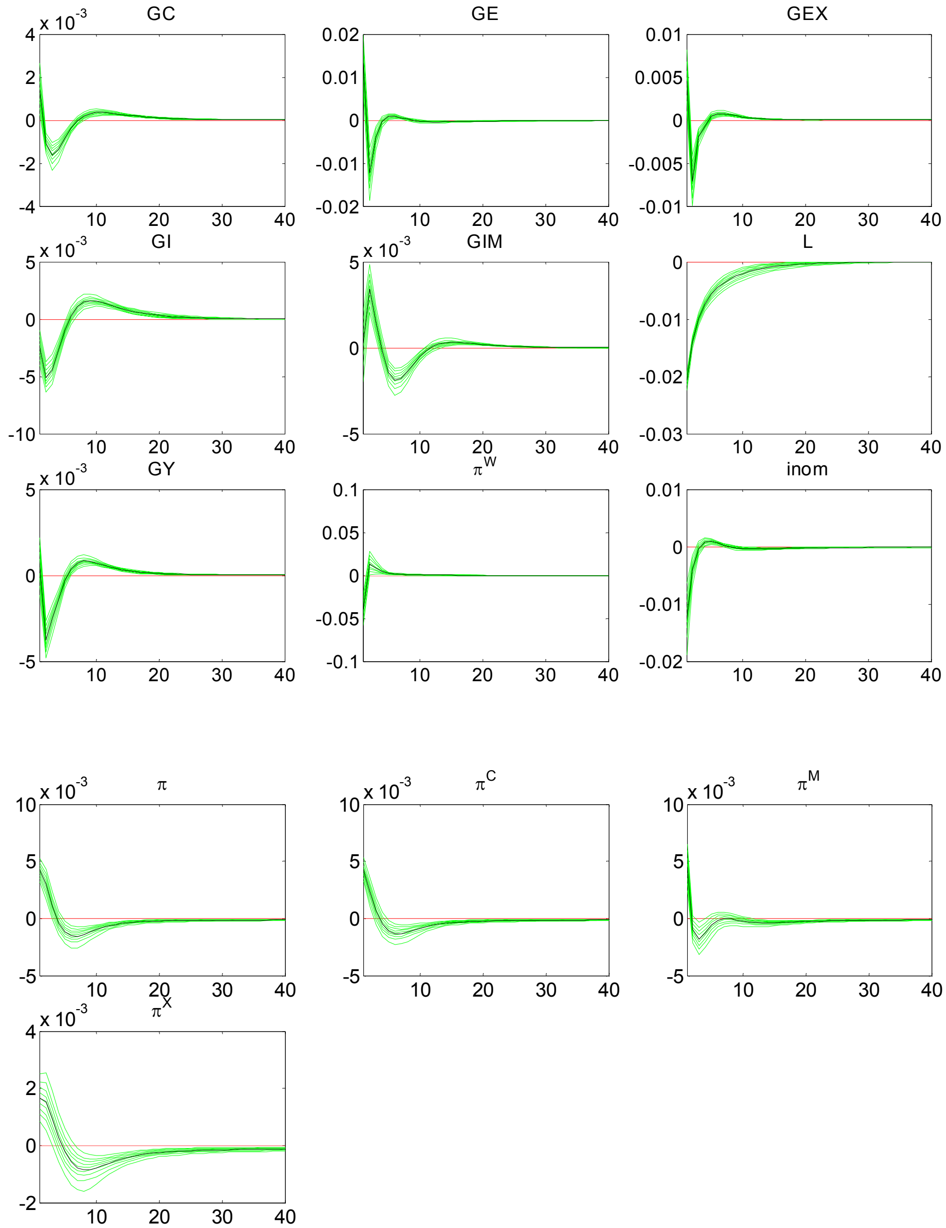

Figure 55. Unit shock to ε^{W} **.**

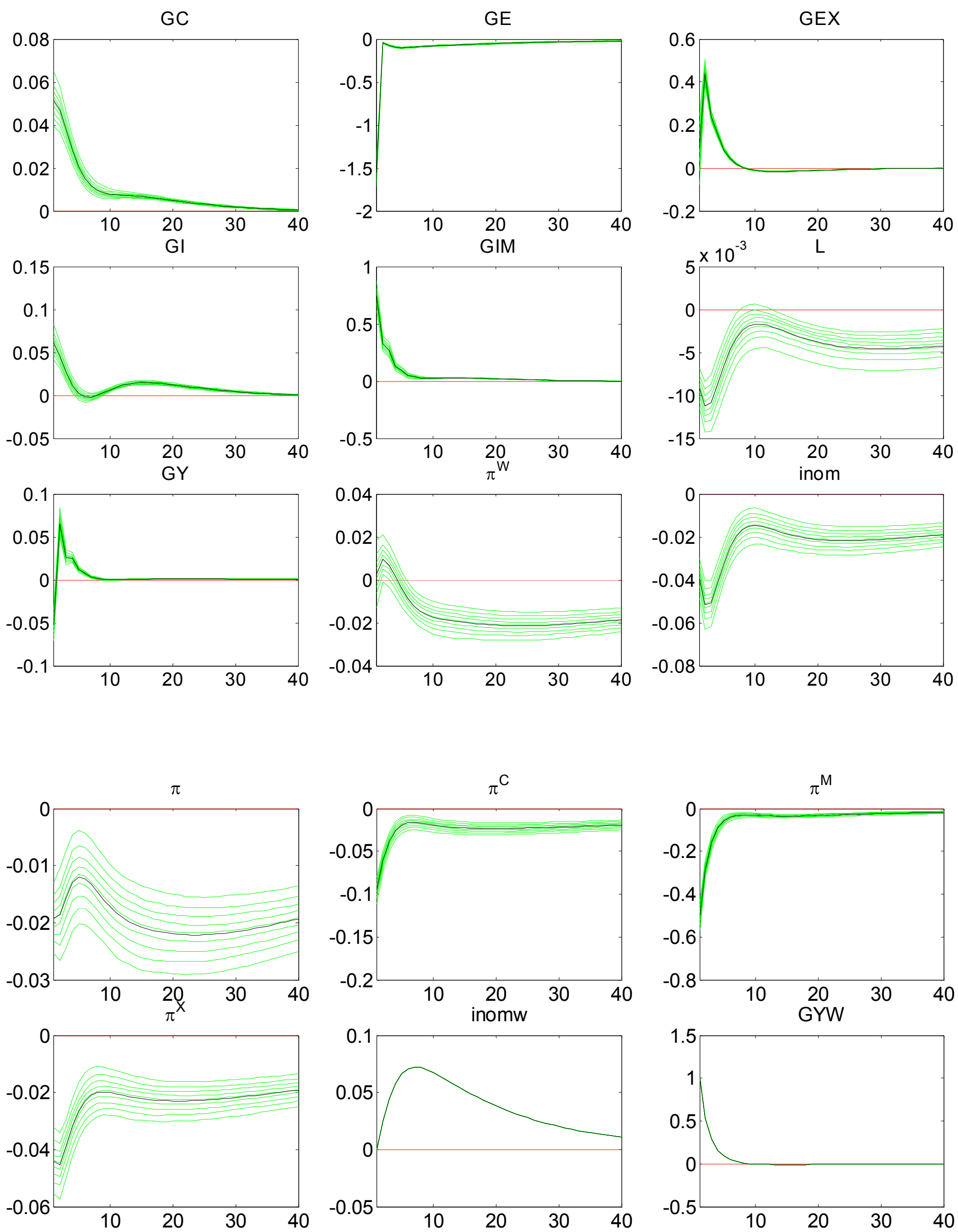

Figure 56. Unit shock to ε^{YW} .

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Abstract

This paper presents an open economy DSGE model, which is estimated on a euro area data set using Bayesian techniques. An attempt is made to impose stochastic assumptions which are consistent with observed trends. In particular we allow for a unit root in technology which allows us to work with actual growth rates. In addition we respect the long run equilibrium constraints implied by the model. The model is compared to a VECM in order to detect weaknesses in the specification. A full Bayesian IRF analysis is performed with a detailed sensitivity analysis of the IRF shape versus model coefficients.

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